Final Dissertation

Study of Architectures and Algorithms for Software Galileo Receivers

Maurizio Fantino

Tutor
prof. Letizia Lo Presti

Co-ordinator of the Research Doctorate Course
prof. Ivo Montrosset

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To all the great people who supported me
Dietro ogni problema
c'è un opportunità

Galileo Galilei
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Abstract

The increased use of digital technology in communications is resulting in more functions of contemporary radio systems that are being implemented using a Software-Defined-Radio approach. The growing interest towards navigation applications, the advent of the European Navigation System Galileo and the new generation GPS, make the SDR approach an interesting perspective to develop a software reconfigurable receiver for positioning applications. Even if the present technology does not allow to totally implement a personal communications receiver, navigation terminals demand for less stringent processing and can be almost completely designed with software defined radio techniques. One of the basic function of a navigation receiver is the signal tracking of a CDMA modulated Signal-In-Space (SIS) broadcast by the satellites constellation (GPS or the future European Galileo).

The goal of the research activity reported in this Ph.D. Dissertation is the study of the receiver architectures and algorithms, which can be successfully implemented by means of software routines on a reconfigurable hardware platform.

The fundamental blocks of a Navigation receiver are the acquisition stage and the tracking loops. For such a reason particular care has been addressed to the strategies able to acquire and consequently track the Galileo L1F signal, which is used to broadcast the Galileo Open Service with the novel BOC(1,1) modulation.

The analysis has been done both from a theoretical point of view and by means of computer simulations in order to prove the models developed. Particular attention has been devoted to the following aspects:

1. the choice of the sampling frequency and its impact on the algorithms performance and on the hardware requirements in terms of system complexity

2. the availability of a pilot channel made by a concatenation of a primary and a secondary codes

3. the particular shape of the BOC(1,1) autocorrelation function, which presents two side lobes
Last but not least, the Quality Control concept is introduced and a modified tracking architecture investigated. An Extended Kalman filter is introduced inside the classical tracking loop with the aim to detect and to monitor the presence of a multipath component affecting the line-of-sight signal.
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<td>ADC</td>
<td>Analog to Digital Converter</td>
</tr>
<tr>
<td>AGC</td>
<td>Automatic Gain Control</td>
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<td>BB</td>
<td>BaseBand</td>
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<td>BOC</td>
<td>Binary Offset Carrier</td>
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<td>BPSK</td>
<td>Biphase Shift Keying</td>
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<tr>
<td>C/A</td>
<td>Clear/Acquisition or Coarse/Acquisition</td>
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<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CS</td>
<td>Commercial Service</td>
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<tr>
<td>ECEF</td>
<td>Earth Centered Earth Fixed</td>
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<td>EKF</td>
<td>Extended Kalman Filter</td>
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<td>EWF</td>
<td>Evil Waveforms</td>
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<td>DLL</td>
<td>Delay Locked Loop</td>
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<td>DSP</td>
<td>Digital Signal Processor</td>
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<td>FFT</td>
<td>Fast Fourier Transform</td>
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<td>FLL</td>
<td>Frequency Locked Loop</td>
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<td>GAS</td>
<td>Governmental Access Service</td>
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<td>GEO</td>
<td>Geostationary</td>
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<td>GLONASS</td>
<td>GLObal NAvigation Satellite System</td>
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<td>GNSS</td>
<td>Global Navigation Satellite System</td>
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<tr>
<td>GPS</td>
<td>Precise Positioning Service</td>
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<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
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<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
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<td>LEO</td>
<td>Low Earth Orbit</td>
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<td>LFSR</td>
<td>Linear Feedback Shift Register</td>
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<td>LNA</td>
<td>Low Noise Amplifier</td>
</tr>
<tr>
<td>LOS</td>
<td>Line Of Sight</td>
</tr>
<tr>
<td>OAS</td>
<td>Open Access Service</td>
</tr>
<tr>
<td>OS</td>
<td>Open Service</td>
</tr>
<tr>
<td>P</td>
<td>Precise</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase Locked Loop</td>
</tr>
<tr>
<td>PRN</td>
<td>Pseudo Random Noise</td>
</tr>
<tr>
<td>PRS</td>
<td>Public Regulated Service</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>RHCP</td>
<td>Right-Hand Circular Polarized</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>SAS</td>
<td>Safety Access Service</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------</td>
</tr>
<tr>
<td>SIS</td>
<td>Signal In Space</td>
</tr>
<tr>
<td>SISA</td>
<td>Signal In Space Accuracy</td>
</tr>
<tr>
<td>SMF</td>
<td>Segmented matched filter</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SOL</td>
<td>Security of Life</td>
</tr>
<tr>
<td>SPS</td>
<td>Standard (or Simple) Positioning Service</td>
</tr>
<tr>
<td>TOA</td>
<td>Time Of Arrival</td>
</tr>
</tbody>
</table>
Part I

Navigation and Galileo System Overview
Chapter 1

Introduction

1.1 History of global navigation satellite systems

The need of navigation is as old as the human history. The compass was discovered and used in Chinese wars during foggy weather before recorded history.

Identifying and remembering objects and landmarks as points of reference were the techniques that the early man used to find his way through jungles and deserts. Leaving stones, marking trees, referencing mountains were the early navigational aids. Stones, trees and mountains were the early examples of "points of reference", a concept that has evolved through times with the advent of (and the need for) more sophisticated techniques, objects and instruments.

A better method was to record this spatial information on a clay tablet or piece of parchment which could be copied and handed from one person to another. We call these maps. The first recorded maps date back to the Mesopotamians some 5,000 years ago, constituting a revolution in geographic positioning that has enjoyed widespread use ever since. While the technology behind cartographic techniques has improved many orders of magnitude over the centuries, conceptually they remain fundamentally the same even today.

Ancient civilizations discovered early the usefulness of maritime navigation. In the first times, they traveled using the coastal line as a reference point, otherwise they would have been lost in the sea. It was natural for these civilizations to look up at the starry sky; looking at it and applying human natural curiosity, some of them became expert in astronomy. Maybe the first discovery they made was to notice that all the stars rotate around a fixed point in the sky. In actual time, this point is occupied by the Polar Star (α Ursae Minoris) but it changes position in the sky due to equinox precession and other astronomical phenomena; moreover it is impossible to be seen during daytime.

However, this fixed–point indicates the north direction (in northern hemisphere), because it’s the intersection between Earth’s polar axis and the sky globe, and the observer
latitude. In this way, some populations began to use this system to get oriented in open
sea (and not only), thus allowing them not to be vinculated to the coast visibility and to
reach very distant places from their countries.

Figure 1.1. Changing position of the celestial North Pole.

After millenniums, we still look at the sky to get oriented, not only on the sea, but also
in the air and on the ground. Technology has advanced and from simple “star looking”,
passing by compasses and sextants, we have arrived to use satellite systems to know
where we are.

The first ideas to use a satellite system for global positioning emerged in the Sixties:
the U.S. Department of Defense (DoD) needed a global, all-weather, continuously avail-
able and highly accurate positioning and navigation system. In those years, the U.S.
Navy developed the first two navigation satellite systems [1]:

- Transit, which consisted of 7 satellites in low-altitude polar orbits, that used Doppler
  shift to give two-dimensions position coordinates;

- Timation, which was the first system to use atomic clocks instead of quartz–crystal
  oscillators.

In the same period, the U.S. Air Force developed another project, called 621B.

In December 1973, DoD decided to create a joint program office (JPO) which was in
charge of coordinating the developing of a new global positioning satellite system, whose
characteristics resembled and upgraded those used in the previous projects. In this way,
the NAVSTAR–GPS was born [1]. During all the Seventies there was the development of


1.2 Basics of satellite navigation

GPS and Galileo satellite are systems based on the measurement of the Time of Arrival (TOA) of an electromagnetic signal. Satellites transmit particular codes, called pseudo-random noise (PRN) codes, and user determines its position by evaluating the time that the signal needs to travel from the satellites to the receiver. This is possible thanks to the very accurate atomic clocks on-board the satellites, all synchronized at the same time among them. The receiver clock is hardly synchronized to the system time, but the synchronization with it can be achieved when the signal is acquired and tracked. The navigation message, which contains some useful corrections for the receiver, can be then read and decoded and the receiver can be synchronized. It is assumed that the position of satellites is precisely known [4]. The time at which the signal left the satellite is embedded in the
ranging signal, so the receiver can calculate the propagation time of the signal; multiply-
ing this by the speed of light. Finally, the receiver determines the user–to–satellite range. As a result of this computation, the user would be located on the surface of a sphere centered on the satellite and with a radius equal to the user–to–satellite range measured. Another measure from another satellite causes the user to be located at the intersection of the two spheres, i.e. on the perimeter of the circle resulting by this intersection (see figure 1.2).

![Figure 1.2. Two satellites ranging measurements](image)

Measuring another ranging signal from a third satellite, the user will be located in one of the two points resulting from the intersection between the perimeter and the third sphere; if the user is on Earth, than the correct position will be the one situated below the plane of the satellites.
The lack of synchronization between the system time and the receiver clock introduces the need to add another ranging measurement from a further satellite. This measure is required to determine the three-dimensional position of the user and the receiver clock offset with respect to the system time. Therefore, for a fully determination of the user’s position, at least four satellites in view are required \[4\].

Other causes of errors are present in the range measurement, in addition to the time offset, so what is computed it is generally called pseudorange. However, in the following simple analysis, these errors are omitted, but the receiver clock offset. The user position, represented in the Earth Centered Earth Fixed (ECEF) coordinate system \[4\], can be represented by the vector

\[ u = (x_u, y_u, z_u) \]  

while the satellite position is represented by

\[ s = (x_s, y_s, z_s) \].

The satellite–to–user vector is

\[ r = s - u \]  

and its magnitude is

\[ r = \|r\| = \|s - u\| \].
As previously stated, the distance $r$ is calculated measuring the TOA of the ranging code from satellite to user

$$ r = c(T_u - T_s) = c \Delta t $$

(1.5)

where $T_u$ is the system time at which the signal is received by the user, $T_s$ is the system time at which the signal was broadcast by the satellite and $c$ is the speed of light.

This expression is generally called geometric range, but it is not what the receivers really measure. The receiver and the satellite clocks, in fact, generally have a bias error from the reference time ($t_u$ for the receiver and $\delta t$ for the satellite clock). So the pseudorange $\rho$ (see Figure 1.4) is computed as

$$ \rho = c\left[(T_u + t_u) - (T_s + \delta t)\right] $$

$$ = c (T_u - T_s) + c(t_u - \delta t) $$

$$ = r + c(t_u - \delta t) $$

(1.6)

The satellite ground network uploads to the satellites the correction for the offset $\delta t$, which is then broadcast to the user by the navigation message; in this way, $\delta t$ is no longer considered as unknown [4]. Hence,

$$ \rho = r + c t_u $$

(1.7)

The unknowns to be determined are the user position in three dimensions $(x_u, y_u, z_u)$ and the offset of the receiver clock from system time $t_u$, so at least four pseudorange measurements are required. The equations involved in pseudorange determination are
nonlinear, so they should be linearized using, for example, an approximate user position around which linearize.

1.3 Signal acquisition and tracking overview

The Galileo Signal-In-Space (SIS) is made of navigation data, transmitted using the CDMA technique: the data are multiplied by the proper PRN code; each satellite uses a specific code and all the PRN codes are theoretically orthogonal, that is to say their cross-correlation functions are ideally zero, in practice they assume very low values.

The first operation performed by a receiver when is turned on is to detect which satellites are visible, which is performed using the PRN code cross–correlation diversity. In order to determine the user’s position is then necessary to evaluate the pseudo-ranges and to access the navigation data: the receiver replicates the PRN sequence for the desired satellite vehicle along with the replica carrier signal, including Doppler effects.

The synchronization between the two PRN codes is accomplished in three steps [4]:

1. code phase acquisition process, which reduces the uncertainty interval of the input code phase to less than one code phase chip width,
2. code phase tracking process, which accurately tracks the variation of the incoming code phase and keeps the code phase alignment error within an allowable limit,
3. carrier phase alignment, which performs the carrier recovery and tracking.

The acquisition process in conjunction with code and carrier tracking are the bottleneck of GNSS receiver: besides investigating a great number of possible code phases in order to detect code-phase alignment, it is necessary to look for different carrier Doppler shifts, since the Doppler effect due to the relative motion between the satellite and the user is not negligible and the acquisition process has a non-coherent nature. It is not possible, indeed, to track the carrier frequency since the PRN code delay is not detected, so the two operations have to be performed together. When the code and carrier are synchronized, the navigation message can be demodulated and the receiver can have access to all the data useful for the pseudorange evaluation [4]. Finally, better the code and carrier tracking is performed better is the evaluation of the pseudorange measurements and consequently better is the user position computation.

1.4 Thesis Organization

This thesis is organized in six parts. The first part cope with the basic principles behind a satellite navigation system, the European Galileo project and its signal definition and modulations. The second part is an introduction about the receiver architectures and its
fundamental blocks. In the third part the acquisition strategy which can be successfully implemented in a software Galileo receiver are investigated for the BOC(1,1) modulation, while the fourth part deals with the signal tracking algorithms. The fifth part is the analysis of a Quality Control technique applied to the code tracking block able to detect a single multipath component affecting the received signal. The last part of this work collects the appendix and bibliography.
Chapter 2

Galileo system description

Galileo is the forthcoming European Navigation System. This chapter aims at describing the Galileo architecture and the main services which will be provided by the system.

2.1 System architecture

The Galileo system architecture has been developed to provide various kinds of services to the users, but at the same time particular attention has been devoted to the minimization of the development and operating costs, to the risks inherent in a so unusual and complex project and to the interoperability with existing GNSS systems, such as GPS. The Galileo architecture is principally made of four components [3]:

- Global component
- Regional component
- Local component
- User segment

2.1.1 Global component

The global component comprises the space segment and the ground segment, which are all the necessary infrastructure elements to provide all the Galileo services. It is the Galileo system "core" component.

Space segment

The Galileo Space Segment comprises a constellation of a total of 30 Medium–Earth Orbit (MEO) satellites in a so-called Walker 27/3/1 constellation, where three of them are spares. The satellites include:
• A platform
• A navigation payload
• A Search and Rescue payload.

The satellites will broadcast precise time signals, together with clock synchronization, orbit ephemeris and other data. The Galileo constellation has been optimized to the following nominal constellation specifications [3]:

• Circular orbits with a semi-major axis of 29984 km (which corresponds to 23616 km altitude);
• Orbital inclination of 56°;
• Three equally spaced orbital planes;
• Nine operational satellites, equally spaced in each plane;
• One spare satellite (also transmitting) in each plane.

The maximum number of visible satellites, at any time and at any location on Earth, is calculated to be:

<table>
<thead>
<tr>
<th>Receiver elevation masking angle</th>
<th>Number of visible Galileo satellites</th>
<th>Number of visible GPS satellites</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>5°</td>
<td>13</td>
<td>12</td>
<td>25</td>
</tr>
<tr>
<td>10°</td>
<td>11</td>
<td>10</td>
<td>21</td>
</tr>
<tr>
<td>15°</td>
<td>9</td>
<td>8</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 2.1. Mapping of Galileo Navigation Signals onto Galileo Navigation Services

Ground segment

Two control centers will manage the control functions by a dedicated Ground Control Segment (GCS) and mission functions by a dedicated Ground Mission Segment (GMS).

The spacecraft and constellation maintenance will be in charge of the GCS, which will handle operations such as the orbit determination and time synchronization. GCS will use a global network made of five Telemetry, Telecommand and Tracking TTC stations, using spread spectrum modulations and so providing robust and interference-free operations.

The navigation signal of all the satellites will be monitored on a continuous basis by the GMS, which will use a global network of thirty Galileo Sensor Stations (GSS). Five Up-Link Stations (ULS) will be used for the communication among satellites and GMS. Two are the main purposes of this segment:
1. evaluate the precise satellite orbit and the clock offset, including a forecast of predicted variations, and upload these computations to satellites.

2. check the integrity of the satellite signals. The results of this check are uploaded to selected satellite and broadcast; in this way any user will receive at least two integrity messages.

The Service Center is another component of the Ground segment, which will be the interface between service providers and users. It will be in duty of interfacing Galileo with non–European Regional Components, with Search & Rescue service providers and with the Galileo Commercial Service providers [5].

![Figure 2.1. Global component](image)

### 2.1.2 Regional Component

Regional segment will be composed by ground stations, sensor and up–link stations [3]. For specific area, like particular geographic areas, it will be possible to up–link integrity data calculated by regional ground control stations.

### 2.1.3 Local Component

Galileo will provide high levels of performances to the users, the presence of local elements will increase the system functionalities and capabilities. Radio data links or communication networks, such as GSM or UMTS, may carry further positioning informations, providing local differential correction signals, allowing users to correct the range
signal from the satellite. Moreover, they will allow to receive it even in difficult environment, such as indoor locations and reducing the Time to Alarm with integrity improvement. Local Elements will allow the users to achieve a high level of accuracy, which can be used in several application fields, such as train control, ship control in harbor, aided-landing for airplanes and so on. Last but not least, the local component will be very useful for the Search & Rescue service. A user who requires help can communicate its position to a Service Center, which can inform him about all the rescue progress using integration with mobile communication systems, such as GSM or UMTS [3].

2.1.4 User segment

The user segment is composed by all different kind of receivers. The differences among them is the capability to use different Galileo services, of selecting particular frequency bands, being able to access regional and local components and being interoperable with other GNSS [3].

2.2 System Services

The Galileo system will provide five positioning services using only its satellites, but it can provide other three categories of services resulting from the combined use with other systems.

2.2.1 Satellite-only services

These services are provided by the Global Component and the precision reached depends only on the users’ equipment.

Open Service (OS)

It will provide, free of charge, position, velocity and timing signals. All the mass-market applications, such as car navigation or integration on mobile phones, will get Benedict from this service. It will be also useful in scientific applications since it provides a precise timing service. The OS will be broadcast on two different signals separated in frequency (E5a and L1 – see section 2.3.1) to allow ionospheric error correction by differentiation of ranging measurements from each carrier frequency. Each frequency will use two ranging signals: one of them contains data, while the other, called pilot channel, is data-less, with the aim to obtain more precise ranging measurements. Integrity will not be applied [3].
2.2 – System Services

Commercial Service (CS)

It will add to the OS two encrypted signals (*Multiple Carrier*) of ranging codes and data, providing more precise positioning and timing than OS, and value–added services on payment of a fee; these can consist of precise timing, precise ionospheric delay models and local differential corrections in presence of a local element [3].

Public Regulated Service (PRS)

It is an access–controlled and robust service for governmental applications. The access will be controlled by the European Union Members through the encryption of the signals, the appropriate key distribution and the control of receivers distribution. This service will be used for applications devoted to European and National Security, for civil protection, for some critical industrial and economic applications.

This service has the requirements to be resistant to interference, malicious and accidental jamming [3]. It will use appropriate interference mitigation techniques to provide a very high level of protection against the threats which can affect the OS. The PRS will be broadcast using wide–band signals on separate frequency with respect to the OS. The use of wide–band signals will allow the PRS to be more resistant to involuntary or malicious jamming. Therefore, this service will offer a better continuity of service. Even if the use of this service is protected and regulated, its control is in duty of civil administrations, such as EU members governments.

Safety–of–Life Service (SoL)

It will be used for transport applications where lives can be endangered by a degraded performance of the navigation system, such as in airplanes landing approach, and in areas where traditional ground infrastructure are not available.

For this service three frequency carriers will be used. The high performance level will be assured by the use of two frequency carriers to improve ionospheric error estimation by differentiation of the ranging measurements made at each frequency, transmitting data only on one ranging signal per carrier (the other is only a pilot ranging code for more precise navigation measurements) and broadcasting the integrity data. This service will be offered globally and free of charge; the system will have the capability to authenticate the signal received to assure the users that the received signal is the actual Galileo signal [3].

Two condition of risk exposure will be considered by this service:

- the *Critical Level*, which will cover time critical operations as the aircraft landing;
- the *Non–Critical Level*, will cover operations that are less time critical as the open sea navigation.
Search and Rescue Service

The Search and Rescue service, provided by the COSPAS–SARSAT cooperative will also be supported by Galileo. COSPAS–SARSAT is a satellite system designed to provide distress alert and location data to assist search and rescue (SAR) operations. It was started in 1979 with an international agreement between USA, former USSR, Canada and France. This system consists of four LEO (Low Earth Orbit, in polar orbit) and three GEO (Geo-stationary Earth Orbit) satellites [6].

The employment of Galileo in this cooperation will allow improvements of the existing search and rescue system.

2.2.2 Locally Assisted Services

In order to increase position accuracy, integrity and availability, Local Components will enhance the Galileo Services. Local elements can provide differential corrections, reaching a position accuracy better than 1 meter and an improving of integrity alarm limits. Position determination can be improved using the Three Carriers Ambiguity Resolution (TCAR) with errors below 10 centimeters.

In particular and difficult environment, the availability can be improved by reducing the amount of data transmitted and by using local stations, pseudolites, transmitting satellite-like signals [3].

2.3 System Signal Structure

One of the main objectives in the Galileo signal definition is to assure the compatibility and interoperability between Galileo and other GNSS system, in particular with GPS. At the same time, Galileo must be an independent system avoiding or reducing the possibilities of simultaneous failure of all GNSS. Galileo should not interfere with other services that operate in the same RF spectrum portion [7]. All these problematic aspects are taken into account by the Signal Task Force, which is the organization in charge of the Galileo signal definition and future modernization.

2.3.1 Frequency and Signal Baseline

Galileo system will use three frequency ranges: 1164 – 1215 MHz (E5a and E5b), 1260 – 1300 MHz (E6) and 1559 – 1592 MHz (E2–L1–E1), which are part of the Radio Navigation Satellite Service (RNSS) allocation (see Figure 2.2).

All satellites will use the same frequencies because the Code Division Multiple Access (CDMA) will be used, to be compatible with the GPS approach. In the same way, some of the Galileo frequencies are the same of GPS: low interference will be achieved by using different modulations and coding schemes. Galileo will provide ten navigation signals in
2.3 – System Signal Structure

*Right Hand Circular Polarization* (RHCP): six signals will be accessible to all users, for OS and SoL services, on the E5a, E5b, E2–L1–E1 carriers (including three data–less channels, also said *pilot channels*); two encrypted signals on E6 accessible to users through a given CS provider; two signals (one in E6 and one in E2–L1–E1) with encrypted ranging codes and data accessible only to authorized users of the PRS [8].

![Galileo frequency spectrum](image)

Figure 2.2. Galileo frequency spectrum

Galileo transmitted bandwidth and center frequencies are specified in the Table 2.2.

<table>
<thead>
<tr>
<th>Frequency Band</th>
<th>Carrier Frequency</th>
<th>Transmitted Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>E5a/L5 Band</td>
<td>1176.450 MHz</td>
<td></td>
</tr>
<tr>
<td>E5b Band</td>
<td>1207.140 MHz</td>
<td></td>
</tr>
<tr>
<td>E5 Band (E5a+E5b)</td>
<td>1191.795 MHz</td>
<td>90 × 1.023 MHz</td>
</tr>
<tr>
<td>E6 Band</td>
<td>1278.75 MHz</td>
<td>40 × 1.023 MHz</td>
</tr>
<tr>
<td>E2-L1-E1 Band</td>
<td>1575.42 MHz</td>
<td>40 × 1.023 MHz</td>
</tr>
</tbody>
</table>

Table 2.2. Transmitted Bandwidth and center Frequency for Galileo
Chapter 3

Galileo Signal in Space Overview

In this chapter the main features of the Galileo Signal–in–space will be presented and, in particular, the attention will be focused on the L1F signal, that will be freely accessible to all users and that will be considered for the analysis of the algorithm feasible for a software implementation in a mass–market receiver. This choice is determined also by the "fully compatibility" of Galileo receivers with GPS. The L1 Galileo center frequency is the same as GPS Simple Positioning Service (1575.42 MHz), so the signal transmitted on this frequency are the most useful for the realization of such receivers.

3.1 Galileo Navigation Signal Description

Six Navigation Signals are transmitted by each satellite, which are named L1F, L1P, E6C, E6P, E5a, and E5b signals [9]

- **L1F Signal**: L1F is an open access signal made of a data channel and a pilot channel. The L1F signal corresponds to a I/Nav message type since it contains integrity message and a partially data encryption used just for commercial purposes.

- **L1P Signal**: The L1P signal is a restricted access signal. Its ranging codes and navigation data are both encrypted and it corresponds to a G/Nav message type.

- **E6C Signal**: E6C is a commercial signal which includes a data channel and a pilot channel. Its ranging and navigation data are encrypted and its data stream corresponds to a C/Nav message type.

- **E6P Signals**: The E6P signal is a restricted access signal. Both the ranging codes and navigation data are encrypted. The E6P corresponds to a G/Nav message type.

- **E5a Signal**: The E5a Signal is an open access signal. It includes both data and pilot channels. Its ranging codes and navigation data are unencrypted and accessible to all users. The E5a navigation data stream corresponds to a F/Nav message type.
• **E5b Signal**: The E5b is again an open access signal transmitted in the E5 band and which comprises a data and pilot channels used to transmit unencrypted ranging codes and navigation data. The E5b navigation data stream contains integrity and encrypted commercial data and it corresponds to an I/Nav message type.

The main signal characteristic are reported in Table 3.1.

<table>
<thead>
<tr>
<th>Galileo Signals</th>
<th>RF Channels</th>
<th>Nav. Message type</th>
<th>Description</th>
<th>Ranging Code Encryption</th>
<th>Data Encryption</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1F Signal</td>
<td>L1-B</td>
<td>I/Nav</td>
<td>Open access</td>
<td>No</td>
<td>Partial</td>
</tr>
<tr>
<td>L1P Signal</td>
<td>L1-A</td>
<td>G/Nav</td>
<td>Restricted access code</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>E6C Signal</td>
<td>E6-B</td>
<td>C/Nav</td>
<td>Controlled access code</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>E6P Signal</td>
<td>E6-A</td>
<td>G/Nav</td>
<td>Restricted access code</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>E5a</td>
<td>E5a-I</td>
<td>F/Nav</td>
<td>Open access code</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>E5b</td>
<td>E5b-I</td>
<td>I/Nav</td>
<td>Open access code</td>
<td>No</td>
<td>Partial</td>
</tr>
</tbody>
</table>

| E5 Signal       | Combination of E5a and E5b signals |

Table 3.1. Summary Characteristics of the Galileo Navigation Signals

### 3.2 BOC Modulations

The STF has to choose the modulation scheme for each carrier considering the fulfillment of various features, thus making a compromise among the following [7]:

- minimization of the level of interference caused by Galileo signals in GPS receivers;
- minimization of the implementation losses in the Galileo satellites;
- maximization of the power efficiency in Galileo satellites;
- optimization of the performance and associated complexity of future Galileo user’s receivers.

*Binary Offset Carrier (BOC)*, also called split spectrum signal, and the *Alternate BOC* (AltBOC) modulations has been proposed in order to limit the interference with other signals, while improving the acquisition and tracking performance. A BOC modulation, as explained in [8], is a square sub-carrier modulation in which a spreading code of rate $f_c$ Mchip/s is multiplied by a rectangular sub-carrier of the frequency $f_s$ MHz, which
splits the spectrum of the signal into two parts located at the left and right side of the carrier frequency.

BOC type signals are usually expressed in the form $BOC(f_s, f_c)$ where frequencies are indicated as multiple of 1.023 MHz. For example, a $BOC(10,5)$ signal has a sub-carrier frequency of $10 \times 1.023$ MHz = 10.230 MHz and a chip rate of $5 \times 1.203$ MHz = 5.115 MHz. BOC generally indicates a sine shaped sub-carrier, in other words a sub-carrier function of code-chips according to $\text{sign}(\sin(2\pi f_s t))$, with sub-carrier frequency $f_s$ and a code-chip starting at $t = 0$. $BOC_c$, is a sub-carrier function of code-chips according to $\text{sign}(\cos(2\pi f_s t))$ and it is called BOC–cosine, with sub-carrier frequency $f_s$ and code-chip starting at $t = 0$.

As an example, $BOC(1,1)$ modulation, which is the modulation designed for the L1 channel, uses a 1.023 MHz square wave sub-carrier that modulates a spreading code at a rate of 1.023 Mchip/s. The spreading code transitions are aligned with the transitions of the square wave sub-carrier. In Figure 3.1 an example of $BOC(1,1)$ is depicted: the solid line represents the original PRN code, while the dashed line is the $BOC(1,1)$ modulation after the multiplication with the square wave sub-carrier: each chip of the spreading sequence is applied to one cycle of the square wave.

The AltBOC modulation is derived from the standard BOC: the baseband signal is multiplied with a complex square sub-carrier in order to produce an 8-PSK like constant envelope constellation. The signal so obtained has a BOC shaped power spectrum where the two side lobes can carry different information. This modulation is going to be used just in the E5a–E5b frequency band, achieving low correlation losses, gain in precision due to the possibility to transmit many side-lobes in a wide-band coherent signal, advantages in user receivers performance because simple receivers can use a single band while more complex receivers may operate in a single band mode or in dual band mode, flexibility in term of services definition since a service can use one or both the two bands [8]

### 3.3 Modulation schemes

Signals on the E5 carrier can be used in two different ways: the first uses two QPSK (Quadrature Phase Shifting Keying) signals (one for each of the two bands of E5, i.e. E5a and E5b), while, according to the second, E5a and E5b signals are multiplexed using an AltBOC(15,10) modulation (see Section 3.2), a modulation scheme based on the standard Binary Offset Carrier BOC [9, 10].

The E6 frequency band contains three channels that transmits at the same carrier frequency. Channel A is modulated by a $BOC(10,5)$ signal, while channel B and C are both modulated by a BPSK(5) (Binary Phase Shifting Keying). The first two channels carry navigation data, while channel C is only a pilot channel. The multiplexing scheme to put
together the three channels has not been chosen yet; the investigated solutions is the CASM modulation \[10\].

The E2–L1–E1 frequency band presents the same problems of E6, because there are also three channels transmitted at the same carrier frequency. Channel A is modulated with a cos–phased BOC(15,2.5), while the other two channels use a BOC(1,1) signal. The first two channels carry data, while channel C carries simply a pilot signal \[7\]. The three components of this frequency band are multiplexed using a CASM modulation that ensures a constant envelope of the transmitted signal \[9, 10\].

The main Galileo modulation parameters are briefly summarized in Table 3.2.

![Figure 3.1. Example of PRN code sequence (solid line) and BOC(1,1) signal (dashed line)](image)

<table>
<thead>
<tr>
<th>Frequency bands</th>
<th>E5a</th>
<th>E5b</th>
<th>E6</th>
<th>E2–L1–E1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central frequency</td>
<td>1176.45 MHz</td>
<td>1207.14 MHz</td>
<td>1278.75 MHz</td>
<td>1575.42 MHz</td>
</tr>
<tr>
<td>Modulation type</td>
<td>QPSK(10)</td>
<td>QPSK(10)</td>
<td>BOC(10,5)</td>
<td>BPSK(5)</td>
</tr>
<tr>
<td>Code type</td>
<td>AHRB(15,10)</td>
<td>cosBOC(15,2.5)</td>
<td>BPSK(5)</td>
<td></td>
</tr>
<tr>
<td>Code length</td>
<td>10230 chip</td>
<td>4115 chip</td>
<td>Restricted</td>
<td></td>
</tr>
<tr>
<td>Code rates</td>
<td>10.23 MHz</td>
<td>4115 chip</td>
<td>2.595 MHz</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2. Main Galileo signal parameters
3.4 Galileo L1 Signal

In this thesis the Galileo L1F signal will be considered and then a brief explanation of the main characteristic of the signal broadcast on the L1 center frequency will be addressed in this and in the following sections. However, more detail on the Galileo L1 signal in space definition can be found in [10].

The Galileo satellites transmit the Navigation Signal on the L1 carrier frequency by means of a right-hand circular polarization. The signal at the output of the satellite is shaped with a filter bandwidth of 40.92 MHz. The main L1F signal characteristics are reported in Table 3.3.

<table>
<thead>
<tr>
<th>Signal Parameter</th>
<th>Carrier Frequency</th>
<th>Polarization</th>
<th>Transmitted bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>$f_{L1}$</td>
<td>1575.420 MHz</td>
<td>40.92 MHz</td>
</tr>
</tbody>
</table>

Table 3.3. L1 Frequency Plan

3.4.1 L1 Channel Signal and Modulation

As stated in section 3.1, the Galileo L1 signal is obtained broadcasting the result of the multiplexing of three signal components:

- The L1–A Public Regulated Signal.
- The L1–B data channel, which is the result of the combination of the L1B navigation data stream and the L1B channel PRN code both modulated by the L1B sub–carrier.
- The L1–C channel, which is the pilot channel of the L1F navigation signal modulated by the L1–C sub–carrier.

The three signal components on the L1 carrier are multiplexed together using a CASM or modified Hexaphase modulation [10]. This modulation has been selected since it can ensure a constant envelope of the transmitted signal, and then it can assure better performance of the satellites power amplifiers.

3.4.2 L1F Signal Rates and Ranging Codes

The L1 signal uses ranging code with the chip– and sub–carrier rate of 1.023 MHz. The sub–carrier used is a BOC(1,1), as stated in Table 3.4.

As reported in Table 3.5, the coded and interleaved navigation data streams are transmitted at a rate of 250 symbols per second.
### Table 3.4. L1 chip- and sub-carrier rates

<table>
<thead>
<tr>
<th>Channel</th>
<th>Sub-carrier Type</th>
<th>Sub-carrier Rate</th>
<th>Ranging Code Chip-Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>BOC</td>
<td>1.023</td>
<td>1.023</td>
</tr>
<tr>
<td>C</td>
<td>BOC</td>
<td>1.023</td>
<td>1.023</td>
</tr>
</tbody>
</table>

Table 3.5. L1 channels data rates

<table>
<thead>
<tr>
<th>Channel</th>
<th>Data rate (symbols per second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1F-data</td>
<td>250</td>
</tr>
<tr>
<td>L1F-pilot</td>
<td>Pilot Channel</td>
</tr>
</tbody>
</table>

Many of the receiver functionalities stem from the PRN code autocorrelation function properties, then the choice of the parameters referring to the code sequences are extremely important. These code may be based on Linear Feedback Shift Register LFSR theory.

The ranging code used for the L1F pilot channel is based on the so called tired codes. Tired codes are built modulating a short duration primary code by a long duration secondary code. L1 Coded have the characteristics reported in Table 3.6

<table>
<thead>
<tr>
<th>Channel</th>
<th>Primary code length (chips)</th>
<th>Secondary code length (chips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1F–B</td>
<td>4092</td>
<td>——</td>
</tr>
<tr>
<td>L1F–C</td>
<td>4092</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 3.6. Code lengths for L1 channels

### 3.5 PRN code properties

Orthogonal codes were chosen as spreading sequences of Galileo signals because of their cross-correlation and autocorrelation properties. The autocorrelation characteristics are fundamental to the signal demodulation process.

The autocorrelation function of a PRN sequence $c_i(t)$ characterized with an amplitude $\pm 1$ and chip duration of $T_c$ and period $NT_c$, can be written in the form

$$R_{L1}(\tau) = \frac{1}{NT_c} \int_{-\infty}^{\infty} c_i(t)c_i(t + \tau)dt$$  \hspace{1cm} (3.1)

where $c_i$ is the generic PRN code for the $i^{th}$ satellite and $\tau$ is the time shift.

Reference [4] derives a simplified expression of the autocorrelation function of a generic
3.5 – PRN code properties

PRN sequence

\[ R_{\text{PRN}}(\tau) = -\frac{1}{N} + \frac{N+1}{N} R(\tau) \otimes \sum_{m=-\infty}^{\infty} \delta(\tau - mNT_c) \]  

(3.2)

where \( \delta(t) \) is the Dirac delta function, \( T_c \) is the chipping period, \( NT_c \) is the code period, \( N \) is the PRN code length and \( R(\tau) \) is the triangle function defined by

\[ R(\tau) = \begin{cases} 
1 - \frac{|\tau|}{T_c} & \text{for } |\tau| \leq T_c \\
0 & \text{elsewhere}
\end{cases} \]

The autocorrelation function is shown in Figure 3.2. It is a periodic function with correlation peaks that repeat every code period, the correlation interval is two chips and outside the correlation interval its value is \(-1/N\). This is just a simplified model of the PRN autocorrelation function since there are shifts which lead to non-zero values between two adjacent correlation peaks.

![Autocorrelation function of a maximum length PRN code](image)

Figure 3.2. Autocorrelation function of a maximum length PRN code

PRN codes are designed in order that the cross-correlation function of any two sequences assumes a very low value for any phase or Doppler shift over the entire code period. The ideal cross-correlation function is defined by

\[ R_{ij}(\tau) = \frac{1}{NT_c} \int_{-\infty}^{\infty} c_i(t)c_j(t + \tau)dt \]  

(3.3)

where \( c_i \) is the L1 code for the \( i \)th satellite and \( L1_j \) is the Galileo code for the \( j \)th satellite with \( i \neq j \). Its value is ideally zero, since two different PRN codes are orthogonal. An example of the cross-correlation function computed over one code period of the Galileo L1 codes for satellites one and three is displayed in Figure 3.3.

The acquisition and the tracking processes are based on the autocorrelation and cross-correlation functions properties of the PRN code. In the acquisition phase a local PRN
code is correlated with the received signal for different Doppler shifts of the local carrier. When a peak in the correlation function overcomes a threshold, the satellite transmitting the locally replicated PRN code is declared acquired. The output of this block is the coarse code alignment along with a first estimate of the residual carrier. The problem of this process is that the signal is buried in the noise floor and under certain conditions false alarms can happen. For what concern the tracking level, when a coarse estimation of the doppler frequency and code delays are known a DLL and a PLL are used to fine track the received signal maximizing the correlation value between the local and incoming signals.

The expression of the PRN sequence power spectral density can be derived from the Fourier transform of Equation (3.2)

\[
S_{\text{PRN}}(f) = \frac{A^2}{N^2} \left( \delta(f) + (N + 1) \sum_{m=-\infty, m \neq 0}^{\infty} \sin^2 \left( \frac{m}{N} \right) \delta \left( f - \frac{m}{NT_c} \right) \right)
\]  \tag{3.4}

where the term \(A\) represents the generic signal amplitude and where

\[
sinc(x) = \frac{\sin(\pi x)}{\pi x}
\]  \tag{3.5}

This is a line spectrum with line spacing of \(1/(NT_c)\) and a DC component of value \(A^2/N^2\),

![Cross-correlation function of Galileo L1 code for satellites 1 and 3](image.jpg)
its envelope is

\[ S_{\text{env}}(f) = A^2 \text{sinc}^2(fT_c) \]  

(3.6)

and it is shown in Figure 3.4.

Once again this is just a simplified expression of the power spectrum of a single Galileo L1 code, where it is possible to see the presence of some small fluctuations in spectral line, so that its envelope is not exactly the \text{sinc} function as reported in Equation (3.5). Using the approximation of Figure 3.4, and considering that most of the signal power is contained in the main lobe of the \text{sinc} function, the single–side bandwidth of the PRN code signal is generally approximated with the first zero of the \text{sinc} function, then

\[ B_s = \frac{1}{T_c} = 1.023 \text{ MHz} \]

3.6 **BOC(1,1) properties**

The effect of the sub–carrier multiplication results, as mentioned before, is the splitting of the main lobe of the original code spectrum. The BOC power spectral density can be
found in [11] and can be expressed as

$$G_{BOC(f_s,f_c)}(f) = f_c \left( \frac{\sin \left( \frac{\pi f}{2f_c} \right) \sin \left( \frac{\pi f}{f_c} \right)}{\pi f \cos \left( \frac{\pi f}{f_c} \right)} \right)^2$$  \hspace{1cm} (3.7)

and it is shown in Figure 3.5. The same figure shows the comparison with the power spectrum of GPS C/A code. In the GPS case the most of the signal power is hold in the main lobe since no sub-carrier is applied, and consequently the approximated single-sided bandwidth is $B_s = 1.023$ MHz. For the Galileo case most of the signal power is contained in the two main lobes around the central frequency and the single-sided bandwidth can be then approximated to

$$B_s = \frac{1}{T_c/2} = \frac{2}{T_c} = 2.046$$ MHz

![Galileo BOC(1,1) and GPS C/A code power spectra](image)

**Figure 3.5.** Power spectral density of Galileo BOC(1,1) code (solid line) and comparison with GPS C/A code (dashed line)

$BOC(f_s,f_c)$ modulation presents features allowing acquisition and tracking improvement opening different implementation strategies in the demodulation chain. A better resolution in the time domain for the correlation peak identification stems from the higher rates at which the transition occurs, so that a more precise acquisition phase can theoretically be obtained. In the case of BOC(1,1) one period of the square wave is contained in a single chip duration. The resulting code has a rate which is twice the original
PRN code rate, so if the signal sampling is performed at a rate of two samples per slot\(^1\), the number of samples per chip become four.

While the autocorrelation function of GPS C/A code is triangular like function, the autocorrelation function of Galileo BOC\((f_s,f_c)\) code presents several local maximums. For the GPS case there is no ambiguity regarding its maximum, this problem is present dealing with the Galileo signals. Figure 3.6 shows the autocorrelation function of BOC(1,1) code sampled at four samples per chip of the PRN code and Figure 3.7 is a zoom on the correlation peak.

![Autocorrelation function of Galileo BOC(1,1)](image.png)

**Figure 3.6.** Autocorrelation function of Galileo BOC(1,1)

Figure 3.8 shows a comparison between the autocorrelation functions envelope of GPS C/A code and Galileo BOC(1,1) code around the correlation peak. Both the GPS and the Galileo code are represented with four samples per chip.

A simplified expression of the Galileo BOC(1,1) autocorrelation function is derived in Appendix A.

\(^1\)Galileo code chips are further modulated by a squared sub-carrier, in this thesis it is commonly referred as slot the width of the sub-carrier chip
Figure 3.7. Autocorrelation function of Galileo BOC(1,1) (zoom on the correlation peak)

Figure 3.8. Galileo BOC(1,1) and GPS C/A autocorrelation function envelope comparison
Part II

Receiver Technology
Chapter 4

Channel modeling

In the design process of a GNSS receiver the analysis of the causes of noise and interference and the properties of the propagation channel are of extremely importance.

The primary cause of interference comes from the additive gaussian noise, which can be taken into account both at the acquisition and tracking level. For such a reason, the signal–to–noise ratio values will be derived in this chapter with particular care since they are directly related to the system performance.

Furthermore, in a navigation system it cannot be neglected the effect of the Doppler shift, so that its effect on the carrier frequency and on the spreading code will be analyzed.

Multipath is one of the most common cause of signal interference and performance losses due to the propagation environment. Buildings, trees and any kind of obstacle between the space vehicles and user can be the source of signal reflections intercepted by the receiver antenna. A simple model of the multipath affected channel will be introduced with particular care to the discrete case.

4.1 Causes of noise and interference

The channel link between the satellite and the receiver is usually modeled as an Additive Gaussian channel, so that the received waveform can be written in the form

\[ r(t) = r_C(t) + n_w(t) = A_{\text{IN}} x_{\text{IN}}(t + T_d) \cos [2\pi(f_c + f_D)t + \phi] + n_w(t) \] (4.1)

where

- \( A_{\text{IN}} \) is the signal amplitude
- \( x_{\text{IN}}(t) \) is the received PRN code \( c_{\text{IN}}(t) \) modulated by the square sub–carrier modulation: \( \text{sign}(\sin(2\pi f_{\text{sub}}t)) \)
- \( n_w(t) \) is the Gaussian noise
\( f_c \) is the L1 carrier equal to 1575.42 MHz

\( f_D \) is the Doppler frequency shift

\( T_d \) is the received code time displacement with respect to the local replica of the PRN sequence

Other causes of interference on a correct signal are the signals received from other satellites, but due to the orthogonality of the Galileo PRN codes they are almost indistinguishable from noise, since the signal–to–noise ratio at the input of the digital part of the receiver is lower than \(-20\) dB as it will be explained in Section 4.2.

Multipath, that is particularly present in urban environments may lead a GNSS receiver to fail in its operation to detect the correct signal. Moreover it may cause the receiver to track a reflected signal instead of the real one. This cause of interference is generally studied and faced in the tracking loop design or by the navigation processing. The power of the reflected signal, besides, is reasonably lower than the power of the correct signal, so it is expected that most of the multipath signals are discarded by the acquisition system under normal conditions.

Generally the IF filter bandwidth and the sampling frequency are selected according to the Nyquist Criterion and for this reason the noise affecting the receiver can be assumed to be an additive white gaussian noise. Only the additive white gaussian noise is considered in the analysis and design of the algorithms considered in the thesis, so that the model of the analog signal at the receiver input will be the one indicated by Equation (4.1).

### 4.2 Signal–to–noise ratios at the input of the receiver antenna

The minimum received power for the Galileo code modulated on the L1 band under normal condition, as shown in reference [10], is

\[
S = -157 \text{ dBW} = -127 \text{ dBm} = 10^{-12.7} \text{ mW}
\]

with a thermal noise power spectral density at room temperature of about

\[
N_0 = -170 \text{ dBm/Hz}
\]

The L1F code signal has a double–sided bandwidth of about 4 MHz (see Figure 3.5), so the noise power under the specified conditions is

\[
N = -170 + 10\log(2 \times 10^6) \simeq -107 \text{ dBm}
\]
and the signal–to–noise ratio at the receiver antenna can be computed, in the worst case, as

\[ \frac{S}{N} \simeq -127 \text{ dBm} + 107 \text{ dBm} \simeq -20 \text{ dB} \]  

(4.2)

In order to determine the value of the SNR at the receiver channel input, it should be necessary to know exactly all the characteristics of the receiver architecture as the antenna gain, the noise figure of the down–conversion stages, filters and so on.

### 4.3 Signal–to–noise ratio at the RF Front–End Output

Figure 4.1 shows the first stage of a GNSS receiver, which is the RF Front–End. An overview of the whole receiver will be presented in Chapter 5.

![Figure 4.1. First stages of a GNSS receiver](image)

The receiver waveform at the ADC output \( x[n] \) can be written as

\[ x[n] = x_C[n] + n_w[n] = A_{\text{IN}} x_{\text{IN}}[n + \theta] \cos [2\pi (F_{\text{IF}} + F_{\text{D}}) n + \phi] + n_w[n] \]  

(4.3)

where

- \( A_{\text{IN}} \) is the useful signal amplitude
- \( x_{\text{IN}}[n] \) is the received PRN code modulated by the square sub–carrier
- \( n_w[n] \) is a white gaussian noise with zero mean and variance \( \sigma_n^2 \)
- \( F_{\text{IF}} \) is the normalized intermediate frequency after the down conversions of the RF front–end
- \( F_{\text{D}} \) is the normalized Doppler frequency shift
- \( \theta \) is the normalized delay of the received code with respect to the local replica of the PRN sequence \( x_{\text{LOC}}[n] \)
All these signals and parameters are defined in the discrete–time domain. Therefore, if the sampling frequency of the ADC converter is \( f_s = 1/T_s \), \( F_D \) is defined as

\[
F_D = \frac{f_D}{f_s}
\]

where \( f_D \) is the Doppler shift expressed in Hertz, and

\[
\theta = \frac{T_d}{T_s} = T_d f_s
\]

where \( T_d \) is the delay expressed in seconds of the received code.

In order to define the variance of the noise samples at the RF Front–End output, it is useful to consider the complex envelope of the signal before the ADC, just after the down conversion stages, which can be written in the form

\[
\tilde{x}(t) = A_{IN}x_{IN}(t + T_d) + \tilde{n}_w(t) = A_{IN}x_{IN}(t + T_d) + n_c(t) + jn_s(t)
\]

Figure 4.2 shows the equivalent baseband transfer function of the antialiasing filter in the analog section of the ADC. Before this filter it is meaningless to define the noise variance, since the noise power spectral density is ideally equal to

\[
G_n(f) = N_0/2
\]

over an infinite bandwidth.

The power spectral density of the in–phase and quadrature–phase parts of the noise are respectively

\[
G_{n_c}(f) = G_{n_q}(f) = N_0
\]
4.3 – Signal–to–noise ratio at the RF Front–End Output

Assuming the single–sided bandwidth of the antialiasing filter is equal to $B_s = f_s/2 = 1/(2T_s)$, the variance of the noise in–phase and quadrature–phase samples at the ADC output is

$$\sigma^2_{n_c} = \sigma^2_{n_s} = 2B_s N_0 = \frac{N_0}{T_s}$$

and, since the noise is zero mean, the noise variance is equal to

$$\sigma^2_n = \mathbb{E}\{|n_w[n]|^2\} = \mathbb{E}\{|n_c[n]|^2\} = \mathbb{E}\{|n_s[n]|^2\}$$

where $\mathbb{E}\{x\}$ is the expected value of $x$. The variance value is, therefore

$$\sigma^2_n = 2N_0 B_s$$

The power of the signal component

$$\tilde{x}_C[n] = A_{IN} x_{IN}[n + \theta]$$

can be evaluated as

$$\tilde{P}_C = \lim_{k \to \infty} \frac{1}{k+1} \sum_{n=-k/2}^{k/2} \tilde{x}_C^2[n] = A_{IN}^2$$

The signal–to–noise ratio at the input of the digital section of the receiver can now be defined as follows

$$\text{SNR}_{IN} = \frac{\tilde{P}_C}{2N_0 B_s} = \frac{A_{IN}^2}{\sigma^2_n}$$ \hspace{1cm} (4.4)

from which

$$\text{SNR}_{IN} = \frac{A_{IN}^2}{2N_0 B_s} = \frac{A_{IN}^2}{N_0} \frac{1}{f_s} = \frac{A_{IN}^2}{N_0} \frac{1}{T_s}$$

The expression of the $\text{SNR}_{IN}$ in decibel follows directly from the previous equation:

$$\text{SNR}_{IN} \mid_{\text{dB}} = A_{IN}^2 \frac{\left|\frac{1}{2N_0}\right|_{\text{dB–Hz}} + 10 \log (2T_s)}{N_0} \left|\frac{C}{N_0}\right|_{\text{dB–Hz}} - 10 \log \left(\frac{f_s}{2}\right)$$ \hspace{1cm} (4.5)

where the notation $C = P_c = \tilde{P}_c/2$, which represents the signal power at RF comprising the carrier, is consistent with the definition that can be found in literature (see, for example, the notation used in [4], in [12] or in [13]).

The definition of $\text{SNR}_{IN}$ in Equation (4.4) and the relationship between $\text{SNR}_{IN}$ and $C/N_0$ expressed by Equation (4.5) are coherent with the definitions in reference [13], along with the considered values of the signal–to–noise ratio at the receiver input which have been pointed out in Section 4.2.
4.4 Doppler frequency shift

The Doppler frequency shift effect on the acquisition and tracking stages, caused by the relative motion satellite and user, cannot be neglected. The Doppler dynamic affects both the carrier frequency and the PRN code. The Doppler carrier frequency shift can reach a maximum value of \(\pm 4\) kHz for a low-speed Galileo user, the Doppler effect on the code is of about 2.64 chip/s which can be taken into account in the tracking loop where a fine code alignment is required, but that can be neglected in the acquisition system.

A simplified model for the Galileo orbits is considered for the analysis of the nature of Doppler shift and in order to justify the above quantitative values, as it is commonly done for the GPS system (see also [14]).

With reference to Figure 4.3, each space vehicle is considered to lie on a circular orbit at a medium height from the earth surface \(r_o = 22616\) km, the orbit radius is \(r_s = 29994\) km and the earth radius is \(r_e = 6368\) km.

![Simplified model for the evaluation of the Doppler frequency shift amount](image)

Figure 4.3. Simplified model for the evaluation of the Doppler frequency shift amount

To determine the amount of the Doppler effect, the component of the satellite speed towards the user, \(v_D\), has to be computed, since the Doppler shift on a frequency \(f\) is

\[
\Delta f_D = \pm \frac{f v_D}{c}
\]  

(4.6)

where \(c \approx 3 \times 10^8\) m/s is the light speed.

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4.4 – Doppler frequency shift

Under the assumed hypothesis of a perfect circular orbit, the angular velocity of a satellite can be computed evaluating the centrifugal acceleration needed to maintain the satellite stable at the desired orbit:

\[ a = \frac{g r_e^2}{r_s^2} \]

where \( g \) is the gravitational acceleration at the earth surface. The angular velocity is then

\[ \frac{d\theta}{dt} = \sqrt{\frac{a}{r_s}} = \sqrt{\frac{g r_e^2}{r_s^3}} = 1.214 \times 10^{-4} \text{ rad/s} \]

corresponding to a revolution period of \( T = 14\text{h} 22\text{min} 36.876\text{s} \). The satellite speed is

\[ v_s = \frac{r_s d\theta}{dt} = 29994 \text{km} \times 1.214 \times 10^{-4} \text{ rad/s} = 3641 \text{ m/s} \]

so, with reference to the notation of Figure 4.3, the component of the satellite speed towards the user is equal to

\[ v_D = v_s \sin(\beta) \]

Applying the Carnot and the sine theorem to the triangle \( O \hat{A}S \) of the Figure 4.3, the expression expression of \( \sin(\beta) \) is derived

\[ \sin(\beta) = \frac{r_e \cos(\theta)}{AS} = \frac{r_e \cos(\theta)}{\sqrt{r_e^2 + r_s^2 - 2r_er_s \sin(\theta)}} \]

Substituting Equation (4.9) in (4.8), the expression of \( v_D \) becomes

\[ v_D = \frac{v_s r_e}{AS} \cos(\theta) = \frac{v_s r_e \cos(\theta)}{\sqrt{r_e^2 + r_s^2 - 2r_er_s \sin(\theta)}} \]

and the maximum value can be calculated. From this operation it can be derived the value of \( \theta \) which maximizes \( v_D \)

\[ \theta = \arcsin \left( \frac{r_e}{r_s} \right) = 0.214 \text{ rad} \]

The maximum of the relative speed, with these values of (4.7) and (4.10), is

\[ v_{DMAX} = v_s \frac{r_e}{r_s} = 3641 \text{ m/s} \times \frac{6368 \times 10^3 \text{ m}}{29984 \times 10^3 \text{ m}} \approx 773 \text{ m/s} \]
The value in (4.11) derives from a series of approximations, but it has to be highlighted how with this simple procedure it is possible to determine the range of the Doppler frequency shift. For low-speed users, the maximum amount of the Doppler frequency shift on the L1 carrier can be computed using (4.6):

$$\Delta f_{Dc} = \pm \frac{f_c v_{D\text{MAX}}}{c} = \pm \frac{1575.42 \times 10^6 \text{Hz} \times 773 \text{m/s}}{3 \times 10^8 \text{m/s}} \simeq \pm 4 \text{kHz} \quad (4.12)$$

while the maximum amount of the Doppler shift on the code chipping rate for the Galileo L1F code is

$$\Delta f_{DR} = \pm \frac{R_{CA} v_{D\text{MAX}}}{c} = \pm \frac{1,023 \times 10^6 \text{chip/s} \times 773 \text{m/s}}{3 \times 10^8 \text{m/s}} \simeq \pm 2.64 \text{chip/s} \quad (4.13)$$

The Doppler shift on the code chipping rate can be neglected, as a first order approximation. For the Galileo L1F case the code period is 4 ms and two frequencies different by 2.64 Hz take about 189 ms to change by half a chip, that correspond to 47 code periods. However, if the "fully compatibility" with a GPS receiver is desired, the maximum Doppler carrier frequency shift, which has to be considered is of ±5 kHz, since the GPS satellites have a relative orbit lower than the Galileo ones, then a higher relative speed $v_D$.

### 4.5 Doppler effect on PRN code

In reference [15] is shown how the Doppler affects the PRN code rate. The PRN code rate $R$ of the received signal is equal to

$$R = (1 + \frac{f_D}{f_c}) R_{PRN} \quad (4.14)$$

This change in the code rate does not affect the Acquisition stage, but as already mentioned before in this chapter, its effect cannot be neglected at the code tracking level.

### 4.6 General Multipath Radio Channel Model

Signal multipath occurs when the transmitted signal arrives at the receiver via multiple propagation paths (Figure 4.4). Each of these paths may have separate phase, attenuation, delay and Doppler frequency associated with it. Due to the random phase shift associated with each received signal, they might add up destructively, resulting in a phenomenon called Fading.

Depending on the nature of the multiple paths received, there are two types of multipath channels [16].
4.6.1 Discrete Multipath

When the paths between the transmitter and the receiver are discrete, each with a different attenuation and delay, the channel is called a discrete multipath channel.

As shown in the figure above, the discrete multipath channel can be modeled as follows:

\[
y(t) = \sum_{i=1}^{N} \alpha_i s(t - \tau_i(t)).
\]  

(4.15)

Where

- \( N \) is the number of rays impinging on the receiver,
- \( s(t) \) is the bandpass input signal,
- \( \alpha_i \) is the path attenuation,
- \( \tau_i \) is the path delay.

It can be seen that a natural representation of the discrete multipath channel is a tapped delay line with time varying coefficients and possibly time varying tap spacing. Expressing \( s(t) \) as:

\[
s(t) = \Re \{ \tilde{s}(t)e^{j\pi f_c t} \}
\]

(4.16)
it is possible to write the complex channel output as:

\[ \tilde{y} = \sum_{i=1}^{N} \tilde{\alpha}_i \tilde{s}(t - \tau_i(y)) \]  

(4.17)

where:

- \( f_c \) is the carrier frequency,
- \( \tilde{\alpha}_i = \alpha_i e^{2\pi f_c t} \)

Thus the time varying discrete multipath channel can be described by a time varying complex impulse response:

\[ \tilde{h}(\tau; t) = \sum_{i=1}^{N} \tilde{\alpha}_i \delta(t - \tau_i(t)) \]  

(4.18)

where \( \tilde{\alpha}_i \) is the time varying complex attenuation of each path.

It can already be seen how for a fixed number number of path, \( N \) and path delays \( \tau_i \), the time varying channel is completely characterized by the complex attenuation co-efficients \( \tilde{\alpha}_i \).
Chapter 5

GNSS receiver architecture

In order to understand the global functioning of the system and to properly design the algorithms employed, the general structure of a digital GNSS receiver will be considered in this chapter. A brief presentation of the fundamental receiver blocks will be presented and described.

5.1 GNSS receiver structure

The increased use of digital technology in communications is resulting in more functions of contemporary radio systems that are being implemented using a Software Defined Radio (SDR) approach [17]. The growing interest towards navigation applications, the advent of the European Navigation System Galileo and the new generation GPS, make the SDR approach an interesting perspective to develop a software reconfigurable receiver for positioning applications. Even if the present technology does not allow to totally implement a personal communications receiver, navigation terminals demand for less stringent processing and can be almost completely designed with software defined radio techniques. From a theoretical point of view, the SDR technology represents the possibility to design flexible systems, which can be reprogrammed and thus upgraded almost instantaneously [17, 14]. Such a definition has been adopted in the communications field, where different SDR platforms have been developed in order to prove the concept that once all the analog signals coming through the antenna have been sampled, they can be processed via software routines without using ad–hoc hardware [17]. For such a reason, most of the modern GNSS receivers are digital receivers and in the thesis only digital signal processing will be considered. A general high–level receiver architecture is shown in Figure 5.1.

The Galileo radio–frequency signals is received by a Right–Hand Circular Polarized (RHCP) antenna with a nearly hemispherical gain coverage. It is well known that the computational capabilities of the current hardware platforms do not allow for a direct
digitization of RF signals, and a down-conversion to an intermediate frequency is required, in order to match the processing specifications. These RF signals are amplified by a low–noise pre–amplifier, which determines the noise figure of the receiver. The amplified RF signals are then down–converted to an Intermediate Frequency (IF). In the RF–IF converters the L1 band signals are mixed with the local controlled oscillator signals, then the mixed signals are bandpass filtered into the IF signals. At this point two different solutions are possible: the IF signals are amplified and transformed into Baseband (BB) signals or the signal is processed directly at IF.

The digitized signal is processed by each of the \( N \) digital receiver channels. No de-modulation has taken place, only a signal conversion to the digital baseband frequency. The receiver processing is usually a microprocessor, which performs the baseband functions and the decision–making functions associated to each digital receiver channel.

The three main functions, which are performed by the receiver processor, are acquisition, code tracking and phase alignment. The acquisition process is the estimation of the code pseudorange delay with an uncertainty of less than a chip. The code tracking is the local code fine alignment with respect to the received code and the phase alignment is the carrier recovery.

The parameters the GNSS receiver has to provide to the navigation process are essentially pseudorange time delay, Doppler frequency shift and carrier phase.
5.2 Acquisition methodology and problems

In order to track and successfully decode the information broadcast by the satellite constellation, a GNSS receiver has to employ an acquisition strategy to first detect which satellites are in view. For each satellite, the acquisition has to supply the tracking loops with a coarse estimation of the received code delay, with an uncertainty usually of less that half a chip, and a rough estimation of the Doppler frequency shift.

Sometimes an estimate of the receiver location and the time of the day are available to the receiver, so that it is possible to reduce the acquisition research among a subset of the available satellites. This solution is the so-called warm start. However, when such information are not available the receiver must perform a cold start. All the satellites of the constellation have to be searched for. This process is very time consuming, and to face this problem many receivers perform a parallel search of different satellites.

If the acquisition process is too slow the code delay and Doppler estimate might be out-of-date and the receiver could not be able to track the signals. Therefore the acquisition speed and its complexity are very important in the GNSS receiver design.

5.2.1 The search space

The acquisition stage of a GNSS receiver has to investigate a two-dimensional search space in code delay and Doppler shift for each satellite of the constellation. The code delay is the estimated code delay displacement and the Doppler is the estimate of the Doppler frequency shift of the received carrier and are two independent variables.

Figure 5.2 depicts the search space and shows a grid obtained by digitizing the two variables. Each couple of digitized variables in the grid is referred to as bin and the combination of one Doppler bin and one code bin is a cell.

Since the Galileo BOC(1,1) code are fairly short, then the investigation of all the possible code offset is possible and the code range in the search space is generally the entire code length. The Doppler range depends on the receiver dynamics and also on the receiver oscillator stability. For a low-speed user dynamic the Doppler search range goes from $f_{\text{IF}} - 5\text{kHz}$ and $f_{\text{IF}} + 5\text{kHz}$, where $f_{\text{IF}}$ is the intermediate frequency of the down-converted signal. If the signal is down-converted to BB the Doppler search range is $f_{\text{BB}} - 5\text{kHz}$ and $f_{\text{BB}} + 5\text{kHz}$. These values are analyzed in section 4.4.

The resolution of the code search depends on the uncertainty required by the code tracking section of the receiver and it is generally less than half a chip. The Doppler bin width depends on the maximum frequency resolution inside the pull-in range of the carrier tracking block and by the acquisition scheme properties. In the following chapters the acquisition methodology impact on the Doppler resolution will be investigated in detail. These parameters, and in particular the Doppler frequency steps, have to be set with great care since they are crucial for the acquisition speed.
The selection of a path through the search space depends on the vehicle dynamics, the requirements for the acquisition speed and reliability and the acquisition method chosen. In a sequential search the search pattern is usually with a constant Doppler bin and in the range direction from early to late in order to avoid multipath. The code search strategies have been studied in the articles in reference [18] and [19]. In the Doppler bin direction, the search pattern is typically from the mean value of the Doppler uncertainty and then symmetrically one Doppler bin at a time on either side of this value.

5.3 Signal tracking

When the receiver has acquired the satellite signal, to allow demodulation and decoding of the data message the parameters from the acquisition stage are passed to the tracking loop. Its purpose is to keep the best alignment between received and local codes. As the acquisition process, the tracking stage is also a two–dimensional signal replication process. A general high–level architecture of a navigation tracking channel is show in Figure 5.3.

The receiver tracking loop consists of two loops coupled together. The code tracking
loop is used to maintain the ranging code aligned and the carrier tracking to follow the carrier dynamic due to the Doppler shift. The first is the so called Delay Lock Loop (DLL), while the second consists of a combination of a Frequency Lock Loop (FLL) and a Phase Lock Loop (PLL).

The code loop generates three outputs. An early code, a late code and a prompt code, which are applied to the digitized in–phase (I) and quadrature–phase (Q) input signal. Integrating over one code period or more, three correlations values are used to build a discrimination function used to shift left or right the local replica codes and then maintaining the prompt version aligned to the received ranging code. The prompt code is then used to wipe off the received signal by the ranging code and applied to the carrier loop. By means of a phase detector a traditional FLL or a PLL are used to follow the input carrier shift. The output of the carrier loop is then used for the generation of the I and Q samples used by the code loop.
5.4 Navigation processing

When the signal is acquired and tracked, the bit synchronization can take place and the data demodulation process can start. The navigation message contains the satellite almanac and the ephemeris parameter useful to determine the satellite’s positions. When at least four satellites are tracked it is possible to calculate the user-to-satellite ranges necessary for the evaluation of the user position and velocity using the algorithm described in section 1.2.
Chapter 6

Receiver front–end: analysis of the working frequency and of the number of bits of the ADC

Generally the signal is down–converted from the radio–frequency to an intermediate frequency or to a baseband frequency. This process is performed both to make the signal match the center frequencies and bandwidth of the electronic components and to avoid to process the RF signal directly which will require a higher sampling rate.

The analog–to–digital converter is another important aspect of the GNSS receiver front–end. The receiver performances are determined by the number of bits of the ADC. A higher number of bits can determine a better position accuracy with an increment of the system complexity required in the operations. This is a crucial aspect both for the acquisition and tracking stages, that are the bottle–neck of the entire system.

In this chapter the possible down–conversion methodologies and the number of bits of the ADC will be considered.

6.1 Working frequency

Figure 6.1 presents the different solutions for the operative working frequency of the first receiver stage. Sampling the signal directly at radio–frequency (Figure 6.1(a)) is impractical due to the present technological limits, so this process is generally performed at an intermediate frequency (IF, Figure 6.1(b)). This frequency can be of the order of the Megahertz when the signal is left at an intermediate frequency or of some Kilo赫兹 when the signal is down-converted to the baseband.

The RF signal is not down-converted to the zero center frequency (Figure 6.1(d)), in GNSS applications, generally for three reasons. The first reason is related to the way the down–conversion is made. It is an incoherent process where the RF signal is multiplied
with a local reference sinusoid and then bandpass filtered, the resulting signal is placed to the difference of the two initial frequency. Since the carrier phase is an unknown before the determination of the spreading code and its delay when the difference between the input and local carrier oscillator is zero the phase ambiguity may reduce the signal amplitude or even make it zero. The second reason is the presence of a different Doppler affecting every single channel, which are time varying and dependent to the user dynamic. The last reason is related to an ambiguity in the determination of the Doppler.
frequency shift, as it will be explained in Chapter 8, however this last problem can be eliminated increasing the complexity of the acquisition strategy.

Many different results about the working frequency value and the sampling techniques can be found in literature. In every case, however, the signal is down-converted to an IF or BB frequency. All the references consider a receiver for the GPS C/A code, but the same considerations can be applied to Galileo BOC(1,1) modulation on the L1 carrier. The only difference is the bandwidth of the signal and the sampling rate, but the carrier frequency may remain the same.

6.1.1 Comparison between direct and down-converted digitization

Reference [20] by Tsui and Akos compares two sampling methods. The first after the down-conversion from RF to IF, the second directly at RF according to the software radio concept. The block schemes of the two approaches are shown in Figure 6.2.

![Block scheme for down-converted digitization (a) and RF digitization (b)](image)

More hardware is required in the down-converted digitization scheme since a mixer, two filters and two low-noise amplifiers are needed. However, a better hardware components design can be achieved because the lower center frequency. The direct RF digitization, instead, requires only one filter and one low-noise amplifier, but the bandwidth of the components are wider and more noise goes into the processing chain.

The proposed down-converted digitization, modified to match the Galileo frequencies requirements, uses a center frequency $f_{IF_1} = 22.25$ MHz and a sampling frequency
$f_s = 10 \text{ MHz}$. The signal is under-sampled, but since the double-sided bandwidth of the Galileo BOC(1,1) code is of about 4 MHz, there is no overlapping of the signal replica, as depicted in Figure 6.3. The resulting center frequency is $f_{\text{IF}_2} = 2.25 \text{ MHz}$.

The direct RF digitization should be performed using a sampling frequency $f_s = 9.95677 \text{ MHz}$. In this case the replica bandwidth do not lay on each other and the signal spectrum is down-converted to $f_{\text{IF}_2} = 2.25034 \text{ MHz}$. However, direct RF digitization presents many hardware limitations and down-converted schemes are then more preferable.

The comparison between a down-conversion with no signal overlapping and a digitization with aliasing of the two baseband replicas of the useful signal is shown in Figure 6.4.

The experimental results carried out in the cited article show how the two schemes present the same performances in terms of signal-to-noise ratio after the digitization, but the second allows the possibility to use a lower sampling rate and then a lower processing capability required by the hardware used for the receiver implementation. This particular solution is the one proposed in [13] and its impact on the acquisition stage analyzed on Chapter 8.

### 6.2 AGC and number of bits of the ADC

Together with the down-conversion methods and the selection of the working frequency two other parameters are of extremely importance: the analog-to-digital converter (ADC) and the automatic gain control (AGC) needed to adapt the input signal to the ADC input dynamic.

The AGC controls the ADC input signal amplitude by means of a variable gain amplifier, as depicted in Figure 6.5. The designing problem of the AGC for GNSS application is
that the signal-to-noise ratio at the receiver input is very low. The signal is buried in the noise floor, since it is the composition of a spreading code plus carrier and no demodulation has been performed before the analog-to-digital conversion.

In the 1-bit solution the AGC is useless. The ADC has just to compare the input signal with a zero threshold and set the output bit to "-1" or to "1" when the input is below and
above the threshold respectively. Only a circuit that limits the input signal is required, so that the received signal does not overcome the ADC dynamic, and no AGC is needed.

Reference [20] deals with an 8-bit ADC solution without any AGC but with three amplifiers of 30 dB gain each for a total nominal gain of 90 dB. This solution may be useful when the input signal amplitude is constant but not in an urban environment where the signal amplitude varies rather quickly.

Reference [21] states that an ADC with one, two, three or four output bits is sufficient for GNSS applications. The quantization error of the ADC impacts on the SNR of the GPS receiver and increasing the number of bits reduces the SNR degradation. A 4-bit resolution provides close to minimum SNR degradation and higher resolution offers very little improvements greatly increasing the ADC complexity.

Reference [22] proposes a four-level (2-bit) ADC where the magnitude-bit statistic of the two output bits is exploited in order to set the gain of the variable gain amplifier. In reference [23] the ADC has 4 bits.

In the article in reference [24] a 2-bit ADC is used and the AGC is implemented by observing the ADC output and varying the quantizer thresholds to ensure specific ratios of the output digitized quantities are maintained.

A 2-bit ADC is used in the article in reference [24] and the AGC is implemented by observing the ADC output and varying the quantizer thresholds. In this way the specific ratios of the output digitized quantities are ensured to be maintained.

An interesting comparison between the 2-bit and 1-bit ADC is performed by Persson, Dodds and Bolton in reference [25]. According to the results of [26], a 4-level, i.e., 2-bit, quantization of a Gaussian random variable introduces an additional 12% noise, that is to say distortion, power and 2-level quantization introduces 36% additional noise power. The expected difference between 2-bit and 1-bit ADC is of 24%. In the article [25] simulation results proving these assertions are presented and the authors state they have implemented two versions of an acquisition system for a GPS receiver, the first with a 2-bit ADC, the second with 1-bit ADC. The 1-bit version is preferable to the 2-bit one since it is much less complex and does not require an AGC. Although the 1-bit quantization introduces additional noise, it is relatively insignificant compared with the noise already present in the CDMA signal.

### 6.3 1-bit ADC investigation

The ADC 1-bit solution is very attractive, since no AGC is needed but only an amplifier to limit the input signal. Moreover all the operations can be implemented with a reduced complexity. By the way, the current technology allows to easily implement front-end solution for navigation purposes boarding an ADC which can represent the input signal with a resolution up to 8 bits. If an higher resolution is preferable at the tracking level,
The acquisition block has less stringent requirements an the 1–bit solution can be taken into account.

In the thesis the two level solution will be analyzed to derive the acquisition performance and it will be compared to the case where the received signal is represented with higher resolution. A detailed analysis of the 1–bit solution can be found in reference [15].

6.4 Working frequency influence on the acquisition and tracking stages

The thesis is mainly focused on the acquisition and tracking stages of a GNSS receiver. It will be supposed, for the evaluation of the acquisition and tracking system performances, that the input signal has been down–converted to a proper baseband frequency, whose precise value will be chosen on the basis of the overall channel requirements. Any precise reason for discarding or preferring a particular working frequency arises, as a matter of fact, from the previous analysis of literature results.

Once the frequency value has been chosen, it is possible to use one of the shown methods in order to perform the down–conversion according to the hardware requirements. The signal can be sampled after one or more standard down–conversion stages, or directly at RF, by means of a proper sampling rate that under–samples the RF signal and shifts its spectrum to the desired center frequency. If the obtained number of samples per chip is excessive for the acquisition and tracking systems, a decimation will be performed after a band–pass filtering.

6.4.1 Required sampling rate for the tracking loop

The minimum sampling frequency for a mass–market receiver required by the acquisition stage is usually considered the double of the signal bandwidth, which generally leads to a sampling frequency double of the chipping rate for GPS (see reference [4]) and double of the slot rate for the Galileo BOC(1,1) signal. This is the case presented in [13] and considered in this Thesis. In this way, the synchronization of the received code with respect to the local replica after the acquisition is of half a slot and no complexity is added to the system. The tracking stage, however, cannot obtain a fine synchronization of the two code sequences if the sampling rate is exactly twice the chipping rate as it will be outlined in the following.

In a navigation receiver the real information resides in the chip transitions of the codes, which is used to update the internal counters and, as a consequence, to compute the pseudorange measurements. Dealing with a digital receiver implementation implies to work with signals represented by sequences of samples where those pieces of information are lost. With a proper choice of the sampling frequency this information can be
preserved, and the precise alignment between the incoming and local code can still be obtained by a DLL [27]. It is here recalled that in a Early-Late architecture a discriminator function is built differentiating the early and late correlation functions, and the correlation is the sum of the product among the samples between the received sequence $r(t)$ and the local code $c(t)$.

$$R_N(\tau) = \sum_{k=0}^{N-1} r[k] * c[k + \tau]$$

(6.1)

For example, the use of a sampling frequency of 4.092 MHz for the Galileo BOC(1,1) is not a good choice. In this case each analog chip of the code is perfectly represented by four samples\(^1\). This is the case of Figure 6.6, where two different phase realizations of the same local code with exactly two samples per chip is represented.

\[^1\text{In this analysis the Doppler effect on the incoming is not considered and the incoming signal is considered ideal with no noise, if not differently stated.}\]

---

**Figure 6.6.** Relation between sampling rate and C/A code (ambiguity on distance resolution).
6.4 – Working frequency influence on the acquisition and tracking stages

correlation value and then to the same value of the discrimination function. It is easy to check that a sampling rate of 2 samples per slot gives a time resolution of half a slot or in other words a different correlation value every half slot phase step. The corresponding distance resolution is 73.26 meters, too coarse to obtain an acceptable accuracy on the user position.

It has to be remarked that in a digital receiver it is possible to distinguish between a system clock (at frequency $f_{clk}$) used both for the hardware platform timing and the signal generation, and a processing frequency for the elaboration of the samples obtained by the RF front-end (See Figure 5.3).

Defining $N_s$ as the number of fundamental time steps $\delta t = 1/f_{clk}$ contained in a chip and with $n_d$ the time step between two adjacent samples (processing frequency), the corresponding time and distance resolution can be expressed by:

$$t_{res} = \delta t \cdot \text{g.c.d}(N_s, n_d)$$

$$d_{res} = c \cdot t_{res}$$

where g.c.d is the greatest common divisor function and $c$ is the speed of light, and $t_{res}$ is the smallest value of the phase delay between the incoming and local code, causing a variation at the discriminator output.

It can be seen from Equation (6.2) that the minimum time and distance resolution can be obtained for a certain $\delta t$ when the g.c.d between $N_s$ and $n_d$ is equal to one. This condition corresponds to a sampling frequency not synchronized with the code rate. In such a case, as it is shown in Figure 6.7, two different delayed version of the same sequence give different correlation outputs, so that it is possible to distinguish them and preserve the information about the code transition. Therefore when the DLL aligns the sequence of samples the alignment among the signal transitions is guaranteed. Under this condition, a finer time resolution can be obtained through signal processing, which can be converted into a finer distance resolution.

Figure 6.8 shows how the synchronized rate produces an autocorrelation function which presents some uncertain, flat, regions. The size of the flat region can be derived from Equation 6.2.

Figure 6.9 shows the comparison between the discriminator functions for the L1 code with a BOC(1,1) sub-carrier when a sampling rate synchronized and incommensurable with the code rate is chosen. The presence of the same flat regions, already seen in the autocorrelation function, introduces some uncertainty in the code lock where the DLL could wrongly decide to be locked with the phase of the incoming code and lead to high error in the pseudorange measurements.

An optimum choice of the sampling frequency is then an incommensurable value with respect to the ranging code rate. Under this condition no flat regions are present.
both in the autocorrelation and in the discrimination function, and the best time resolution achievable with the hardware and algorithm implementation can be obtained. Table 6.1 shows how different values of the distance resolution can be obtained through different choices of the system clock and sampling frequency for the Galileo BOC(1,1) signal.

Considering the situation shown in Table 6.1 it can be seen how it is possible to decrease the computational complexity through a proper choice of the sampling rate achieving the better performances with a certain system clock.
6.4 – Working frequency influence on the acquisition and tracking stages

Figure 6.8. Ambiguity length of the autocorrelation function envelope for Galileo BOC(1,1) code.

<table>
<thead>
<tr>
<th>$f_{clk}$ [MHz]</th>
<th>$N_s$</th>
<th>$n_d$</th>
<th>g.c.d</th>
<th>$t_{res}$ [ns]</th>
<th>$d_{res}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>102.3</td>
<td>100</td>
<td>25</td>
<td>25</td>
<td>244.38</td>
<td>73.26</td>
</tr>
<tr>
<td>102.3</td>
<td>100</td>
<td>20</td>
<td>20</td>
<td>195.5</td>
<td>58.60</td>
</tr>
<tr>
<td>102.3</td>
<td>100</td>
<td>23</td>
<td>1</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>204.6</td>
<td>200</td>
<td>50</td>
<td>50</td>
<td>244.38</td>
<td>73.26</td>
</tr>
<tr>
<td>204.6</td>
<td>200</td>
<td>20</td>
<td>20</td>
<td>97.751</td>
<td>29.30</td>
</tr>
<tr>
<td>204.6</td>
<td>200</td>
<td>49</td>
<td>1</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 6.1. Time and distance resolution for different values of the system clock and sampling rate.
Figure 6.9. Discriminator function with a prime sample rate and a synchronized sample rate for the Galileo code.
Part III

Signal Acquisition
Chapter 7

Basic acquisition scheme and correlation methods

The basic acquisition scheme generally employed for the GNSS signal will be presented in this chapter. The different architectures will be analyzed to introduce the acquisition techniques, which are based on the autocorrelation function properties and allow to detect the correct code delay and Doppler frequency shift.

In this chapter the signal is supposed to be down-converted to baseband with a small residual carrier of few Kilohertz $F_{BB}$ as in the case of reference [13]. The considerations herein addressed, however are completely general and the term $F_{BB}$ can be exchanged without loss of generality with $F_{IF}$.

Three different methods to perform the correlation will be described and their main properties and differences will be studied.

7.1 Basic acquisition scheme

The acquisition process is a non–coherent process, since for a GNSS signal is not possible to separate the code from the received signal until the carrier frequency and phase and the Doppler frequency shift are not know. To the other side it is not possible to get the residual carrier from the signal until the code alignment is not recovered.

Figure 7.1 shows a basic non–coherent acquisition scheme, which can be found in references [13] and [28]. This scheme is the so called serial search scheme because it performs a serial search in time delay and Doppler frequency shift domains as depicted in Figure 5.2.

The input signal after the down–conversion stages and the digitization is expressed
by Equation (4.3), which is rewritten here for sake of clarity:

\[ x[n] = x_C[n] + n_w[n] = A_{IN}x_{IN}[n + \theta] \cos[2\pi(F_{BB} + F_D)(n + \theta) + \phi_1] + n_w[n] \quad (7.1) \]

where

- \( A_{IN} \) is the useful signal amplitude
- \( x_{IN}[n] \) is the received PRN code modulated by the square sub–carrier (Galileo)
- \( n_w[n] \) is the gaussian noise with zero mean and variance \( \sigma^2_n \)
- \( F_{BB} \) is the baseband frequency after the down conversions of the RF front–end.
- \( F_D \) is the Doppler frequency shift
- \( \theta \) is the received code delay

The input signal \( x[n] \) is multiplied by the local replica of the PRN code plus sub–carrier \( x_{LOC}[n + \hat{\theta}] \), where \( \hat{\theta} \) is the local code delay. The signal with a non–coherent procedure is split into two branches. In the upper one it is multiplied by a local cosine and in the lower branch it is multiplied by a local sine, whose frequencies are the residual frequency plus the local Doppler frequency shift \( F_{BB} + \hat{F}_D \). Even if the two branches are not the classical in–phase and quadrature–phase branches of a telecommunication system, the same terminology will be adopted to address the branches of the acquisition scheme.
7.1 – Basic acquisition scheme

As already mentioned in chapter 6, the phase \( \phi_1 \) of the received cosine is not known and if the received signal was multiplied by the cosine only, it would be possible to obtain a zero or very small result. The two branches are used in order to not lose part of the signal power, whose level is already very low in comparison to the noise power.

The signals on the two branches can be written as

\[
x_I[n] = A_{IN}x_{IN}[n + \theta] \cos [2\pi(F_{BB} + F_D)(n + \theta) + \phi_1] \cdot A_{LOC}x_{LOC}[n + \hat{\theta}] \cos [2\pi(F_{BB} + \hat{F}_D)(n + \hat{\theta}) + \phi_2]
\]

(7.2)

\[
x_Q[n] = A_{IN}x_{IN}[n + \theta] \cos [2\pi(F_{BB} + F_D)(n + \theta) + \phi_1] \cdot A_{LOC}x_{LOC}[n + \hat{\theta}] \sin [2\pi(F_{BB} + \hat{F}_D)(n + \hat{\theta}) + \phi_2]
\]

(7.3)

where

- \( A_{LOC} \) is the local signal amplitude
- \( x_{LOC}[n] \) is the local replica of the PRN code with the sub–carrier on
- \( \hat{F}_D \) is the local Doppler frequency shift
- \( \hat{\theta} \) is the local code delay

and the noise has been discarded. In the following, the local code will be considered to be equal to the received one and they will be named \( x_C[n] \).

The obtained samples are then summed in the block called, in Figure 7.1, as Average and Dump or Integrate and Dump over one or more code periods to obtained a correlation value. The correlator output for the in–phase branch can be written as

\[
R_I(\theta, \hat{\theta}, F_D, \hat{F}_D) = \sum_{n=0}^{N-1} A_{IN}x_C[n + \theta] \cos [2\pi(F_{BB} + F_D)(n + \theta) + \phi_1] \cdot A_{LOC}x_C[n + \hat{\theta}] \cos [2\pi(F_{BB} + \hat{F}_D)(n + \hat{\theta}) + \phi_2]
\]

(7.4)

and similarly for the quadrature–phase branch

\[
R_Q(\theta, \hat{\theta}, F_D, \hat{F}_D) = \sum_{n=0}^{N-1} A_{IN}x_C[n + \theta] \cos [2\pi(F_{BB} + F_D)(n + \theta) + \phi_1] \cdot A_{LOC}x_C[n + \hat{\theta}] \sin [2\pi(F_{BB} + \hat{F}_D)(n + \hat{\theta}) + \phi_2]
\]

(7.5)
The output of the envelope detector is, therefore

\[
R\left(\theta, \hat{\theta}, F_D, \hat{F}_D\right) = \sqrt{R_I^2\left(\theta, \hat{\theta}, F_D, \hat{F}_D\right) + R_Q^2\left(\theta, \hat{\theta}, F_D, \hat{F}_D\right)}
\] (7.6)

The number of samples \(N\) depends on the sampling rate and on so called \textit{integration time}, which is an integer multiple of the code period. Increasing the integration time the acquisition speed decreases but the system becomes more robust to the additive noise as it will be explained in Chapter 10.

The time required to analyze a cell of the correlation matrix can be fixed or variable and is referred according to the literature as \textit{dwell time}.

From the previous description it follows that the system computes the envelope of the correlation function of the received code with its local replica. The envelope detector output is a matrix with nearly zero values for each bin in which the incoming code and the local one are not aligned in code delay or Doppler frequency shift. The same applies when they are not the same PRN code, which is in line with the consideration carried out in Chapter 3. When the input signal and the local one are the same, some bins, in which the local code results to be aligned with the received signal emerge from the rest of the matrix. In Figure 7.2 an example of the correlation matrix in absence of noise is depicted.

![Figure 7.2. Example of correlation obtained by the basic acquisition system of Figure 7.1](image-url)
In the acquisition process, generally, the scanning of the whole search space is performed using the same input samples. A number of samples corresponding to one or more code periods is stored and all the operations needed to calculate the correlation matrix are carried out with those samples.

### 7.2 Methods for performing the correlation

The bin values in the correlation matrix can be obtained in three different ways: linear correlation, circular correlation and window correlation.

For the analysis of the different methodologies, without losing generality, it is possible to suppose that the residual carrier has been recovered. In this way just the effect of the PRN codes is considered. Naming $y_{IN}$ the PRN received code plus noise and $x_{LOC}$ its local replica, the linear correlation can be expressed as

$$ R[m] = \sum_{n=m}^{N-1} y_{IN}[n+m]x_{LOC}[n] \quad \text{for} \quad m = 0, \ldots, N - 1 \quad (7.7) $$

This operation is shown in Figure 7.3. As $m$ increases by one unit, the correlation is performed with a number of samples equal to the samples at the preceding step minus one, so that, at the last step, the correlation is performed with one sample only. For this reason this method is not generally applied for the computation of the correlation function at the acquisition level.

![Figure 7.3. Linear correlation](image)

The window correlation can be expressed as

$$ R[m] = \sum_{n=0}^{N-1} y_{IN}[n+m]x_{LOC}[n] \quad \text{for} \quad m = 0, \ldots, N - 1 \quad (7.8) $$

and is depicted in Figure 7.4. In this case the number of the received code delays to be acquired is the double of the correlation samples and the local code is shifted over the
received one as \( m \) increases. The number of samples of the correlation is always \( N \).

\[
\begin{array}{cccccccc}
\text{Received Code} & 0 & 1 & \ldots & N-2 & N-1 & 0 & \ldots & N-2 \ N-1 \\
\text{Local Replica} & 0 & 1 & \ldots & N-2 & N-1 & \text{for } n = 0 \\
& 0 & 1 & \ldots & N-2 & N-1 & \text{for } n = 1 \\
& \text{for } n = N-1 \\
\end{array}
\]

Figure 7.4. Window correlation

This correlation can be performed also by swapping the local and the received code sequences. The result is the same since the noise samples of the received code are independent.

The circular correlation is computed over \( N \) samples, as the previous one, but the local code replica is shifted in a circular way over the received code samples. This correlation method is shown in Figure 7.5 and can be expressed through the equation

\[
R[m] = \sum_{n=0}^{N-1} y[n] x[\text{LOC}[(n + m) \mod N]] \quad \text{for } m = 0, \ldots, N-1 \tag{7.9}
\]

where \( \mod \) is the modulus operation.

\[
\begin{array}{cccccccc}
\text{Received Code} & 0 & 1 & \ldots & N-2 & N-1 \\
\text{Local Replica} & 0 & 1 & \ldots & N-2 & N-1 & \text{for } n = 0 \\
& N-1 & 0 & \ldots & N-2 & \text{for } n = 1 \\
& \text{for } n = N-1 \\
\end{array}
\]

Figure 7.5. Circular correlation

Figure 7.6 shows the comparison between two autocorrelation functions obtained performing a circular and a window correlation for the Galileo BOC(1,1) signal for satellite 3. Due to the correlation methodologies and to the code periodicity the two techniques are identically as evinced in the graph comparison.
7.3 – Phase code offset effect

As stated in section 6.4.1, when a sampling frequency not synchronized with the PRN code rate is chosen, the correlation result depends on the code offset positions. Figure 7.7 depicts the correlation comparison between two different input signals with different code delay phase offset and the same local generated code.

Two main particularities can be seen from this comparison. The correlation result is not symmetric anymore with respect to the highest peak, which can be explained taking into account that a not synchronized sampling frequency to the PRN code rate leads to different digitized sequences for different code phase offset. The correlation between two sequences, which are not exactly identical, is not generally an even function. This difference, however, is not particularly meaningful for the implementation of the acquisition system, due to the very low signal–to–noise ratios of the received satellite signal.

Both the linear and the circular correlation can be evaluated by means of DFT operations on the considered digital sequences, appendix B deals with the mathematics behind this approach, which is at the basis of the functioning of the Parallel acquisition in time delay domain scheme introduced in section 9.2.
Figure 7.7. Autocorrelation function of Galileo BOC(1,1) code sampled at a rate $f_s = 2.0578 \times 2 \times R_{BOC}$ for two different code phase alignments.

More important is the reduction of the peak amplitude, which is related to the value of the code phase offset. This introduces a system loss, with respect to the perfectly aligned case, that can be expressed as the ratio between the value of $R_{BOC}(\tau)$ and $R_{BOC}(0)$. Since $R_{BOC}(0) = 1$ the amplitude loss can be expressed just as

$$\alpha_{loss} = |R_{BOC}(\tau)|$$

(7.10)

Since losses are normally expressed in a logarithmic way, and remembering the approximated Galileo BOC(1,1) autocorrelation expression, derived in Appendix A, Equation (7.10) can be written as follows

$$\alpha_{loss}\,|_{\text{dB}} = 20 \log_{10} (R_{BOC}(\tau))$$

$$\simeq 20 \log_{10} \left[ A\left(\frac{\tau}{1/2}\right) - \frac{1}{2} A\left(\frac{\tau - 1/2}{1/2}\right) - \frac{1}{2} A\left(\frac{\tau + 1/2}{1/2}\right) \right]$$

(7.11)

where $A\left(\frac{t}{T}\right)$ is the triangular function defined as

$$A\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T \end{cases}$$
The effect of the correlation amplitude loss and its effect on the acquisition system performance will be further analyzed in Chapter 11.
A plot of this relationship is presented in Figure 7.8

Figure 7.8. Performance loss as a function of the code phase offset
7 – Basic acquisition scheme and correlation methods
Chapter 8

Working frequency and required Doppler frequency steps

In this chapter the serial search scheme will be used to investigate the impact of the selection of the baseband frequency and of the required Doppler frequency step. It will be shown how it is not a good choice to down-convert the signal to the zero frequency and that the Doppler bin width is related to the integration time used to perform the correlation.

8.1 Doppler sign ambiguity in case of zero center frequency

It can be easily shown, using the notation introduced in Chapter 7, how two Doppler frequency shifts with equal absolute value and opposite signs produce the same correlator output when the center frequency is zero. This leads to an ambiguity in the determination of the Doppler which can be overcome only observing the signs of the two correlator branches in the acquisition block.

If $x[n]$ is the digital input signal after the down-conversion stage, the envelope detector output can be written as

$$R(\theta, \hat{\theta}, F_D, \hat{F}_D) = \sqrt{\left\{ \sum_{n=0}^{N-1} x[n]x_{\text{LOC}}[n + \hat{\theta}] \cos \left[ 2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2 \right] \right \}^2 + \left\{ \sum_{n=0}^{N-1} x[n]x_{\text{LOC}}[n + \hat{\theta}] \sin \left[ 2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2 \right] \right \}^2}$$

(8.1)

where $\hat{F}_D$ is the local Doppler frequency shift "guessed estimate".

If the local Doppler frequency shift is $-\hat{F}_D$, reminding that $\cos(\pm \psi) = \cos(\psi)$ and $\sin(\pm \psi) = \pm \sin(\psi)$, the envelope detector output assumes the expression...
\[ R(\theta, \hat{\theta}, F_D, -\hat{F}_D) = \frac{\left\{ \sum_{n=0}^{N-1} x[n]x_{\text{LOC}}[n + \theta] \cos \left[ 2\pi F_D(n + \hat{\theta}) + \phi_2 \right] \right\}^2 + \left\{ -\sum_{n=0}^{N-1} x[n]x_{\text{LOC}}[n + \theta] \sin \left[ 2\pi F_D(n + \hat{\theta}) + \phi_2 \right] \right\}^2}{\left\{ \sum_{n=0}^{N-1} x[n]x_{\text{LOC}}[n + \theta] \cos \left[ 2\pi F_D(n + \hat{\theta}) + \phi_2 \right] \right\}^2} \] (8.2)

It is not possible to discriminate between Equations (8.1) and (8.2) since they are equal. In order to understand the Doppler sign additional information must be considered, such as the sign of the correlator outputs of the in-phase and quadrature-phase branches. This solution, by the way, increases the acquisition system complexity. This is the first reason to discard the RF front-end solution which converts the signal to the zero frequency, the dependence from the carrier phase has already been described in Chapter 6 and a third reason will be shown in the next sections.

### 8.2 Search scheme analysis in the discrete Doppler domain

In order to analyze the behavior of the basic serial search scheme of Figure 7.1 in the discrete Doppler domain, it will be assumed that the local code has been already aligned with the received one, in other words

\[ \hat{\theta} = \theta \]

The general expression of the in-phase and quadrature-phase correlator outputs expressed by Equations (7.4) and (7.5) become

\[ R_1(F_D, \hat{F}_D) = A_{\text{IN}}A_{\text{LOC}} \sum_{n=0}^{N-1} x[n]x_{\text{LOC}}[n + \theta] \cos \left[ 2\pi F_D(n + \theta) \right] \cdot 
\cos \left[ 2\pi \hat{F}_D(n + \theta) \right] \] (8.3)

or equivalently

\[ R_1(F_D, \hat{F}_D) = \frac{A_{\text{IN}}A_{\text{LOC}}}{2} \left\{ \sum_{n=0}^{N-1} \cos \left[ 2\pi (F_D + \hat{F}_D)(n + \theta) + \phi \right] + \sum_{n=0}^{N-1} \cos \left[ 2\pi (F_D - \hat{F}_D)(n + \theta) + \phi \right] \right\} \] (8.4)
where $\phi$ is the difference between the initial phases of the received cosine and of the receiver local oscillator and $F_D$ and $\hat{F}_D$ include both the baseband frequency and the Doppler frequency shift: $F_D = F_{BB} + F_D$.

The quadrature-branch output can be expressed as

$$R_Q(F_D, \hat{F}_D) = A_{IN} A_{LOC} \sum_{n=0}^{N-1} x[n + \theta] \cos [2\pi F_D(n + \theta) + \phi] \cdot \sin [2\pi \hat{F}_D(n + \theta)]. \tag{8.5}$$

or equivalently

$$R_Q(F_D, \hat{F}_D) = \frac{A_{IN} A_{LOC}}{2} \left\{ \sum_{n=0}^{N-1} \sin [2\pi (F_D - \hat{F}_D)(n + \theta) - \phi] + \sum_{n=0}^{N-1} \sin [2\pi (F_D + \hat{F}_D)(n + \theta) + \phi] \right\}. \tag{8.6}$$

Finally the envelope detector output can be written as

$$R(F_D, \hat{F}_D) = \sqrt{R_I^2(F_D, \hat{F}_D) + R_Q^2(F_D, \hat{F}_D)} \tag{8.7}$$

In the following analysis two different serial search implementations will be considered. A system in which the input signal can be considered represented by a floating point value, ideally with an infinite number of bits, and a system in which the input signal is squared or in other words when the input signal can be represented using just a single bit.

### 8.2.1 Envelope detector output in case of code delay and Doppler alignment

The first analysis performed is the study of the maximum of the envelope detector output for various phase difference between the received and local oscillator $\phi$ in the case of a correct code delay ($\hat{\theta} = \theta$) and Doppler frequency step ($\hat{F}_D = F_D$).

An analytical expression of the envelope detector maximum can be written, using the expressions (C.1) and (C.2), derived in appendix C. Equations (8.4) and (8.6) can be written, when code delay and Doppler alignment is perfect, in the form

$$R_I = A_{IN} A_{LOC} \frac{N}{2} \{ D_N (2\pi F_D) \cos [4\pi F_D \theta + 2\pi F_D (N - 1) + \phi] + \cos \phi \} \tag{8.8}$$

for the in-phase branch and

$$R_Q = A_{IN} A_{LOC} \frac{N}{2} \{ D_N (2\pi F_D) \sin [4\pi F_D \theta + 2\pi F_D (N - 1) + \phi] + \sin \phi \} \tag{8.9}$$
for the quadrature-phase branch.

In this and in the following sections, the sampling rate will be considered twice the signal bandwidth. The integration period used for the acquisition is $T = 4\,\text{ms}$ for the Galileo BOC(1,1) modulation, which leads to a total number of samples in the summation equal to about $N = 16400$.

In the specific situation in which

$$\hat{F}_D = F_D = \frac{k}{2N} \quad (8.10)$$

the envelope detector output does not depend on the initial phase $\phi$ due to the fact that an integer value of sinusoidal half periods is contained within an integration interval $T$. In this situation the envelope detector output becomes

$$R = A_{\text{IN}}A_{\text{LOC}}\frac{N}{2} \quad (8.11)$$

For the Galileo BOC(1,1) modulation the code period is 4 ms then the envelope detector output is independent from the initial phase when the Doppler steps is a multiple of 250 Hz. When the signal from the RF front-end is left to the IF the first term in Equation (8.8) and in (8.9) can be filtered out and the dependence from the input phase completely removed. This choice, by the way, requires a higher sampling frequency and then a higher computational complexity is requested by the hardware.

The maximum of the correlation peak for a Doppler frequency step $\Delta f_D = 250\,\text{Hz}$ and for different values of the phase $\phi$ is shown in Figure 8.1 for the floating point input signal and in Figure 8.2 for the 1-bit ADC case. It should be noticed that only the marked values have been calculated, while the solid lines are plotted in order to make the figures intelligible. All the curves, besides, are normalized to the value of Equation (8.11).

The variation of the maximum of the correlation peak with the phase for a zero residual carrier cannot be neglected. This is a reason to avoid the null center frequency. From Figures 8.1 and 8.2, besides, it can be noticed that the peak does not vary with the phase $\phi$ apart from the zero residual carrier, as it was derived in the previous analytical investigation.

The Doppler frequency step generally proposed in literature for GPS, e.g. in reference [4], it is equal to $\Delta f_D = 2/(3T)$, where $T$ is the integration time. If the integration time is one code period the Doppler frequency step for the Galileo BOC(1,1) signal becomes 166.67 Hz. This value can be considered a trade-off between acquisition speed and complexity, phase dependence of the maximum of the correlation peak and the frequency uncertainty accepted by the tracking section of the receiver.

Figures 8.3 and 8.4 show the maximum of the correlation peak for $\Delta f_D = 167\,\text{Hz}$ and for different phase values in the Galileo signal case and they refer, respectively, to the considered implementation of the acquisition system.
8.2 – Search scheme analysis in the discrete Doppler domain

These figures point out the variation with the phase of the maximum of the correlation peak, which decreases as the residual carrier increases and which becomes negligible, for both the implementations, as soon as the frequency is greater than 2 kHz.

8.2.2 Envelope detector output in case variable input Doppler shift and for different initial phases

The previous analysis has considered just the case of a complete code alignment and Doppler frequency shift. In this section the system will be analyzed studying the behavior of the correct cell when the local code is completely aligned with the received code and the Doppler frequency of the received signal falls in between the range of the correct Doppler bin.
By means of a numerical evaluation of Equations (8.4) and (8.6) the graphs of Figures 8.5, 8.6 can be obtained. The envelope detector output has been computed over one code period for the Galileo BOC(1,1) modulation increasing the local search Doppler, with a Doppler frequency step \( \Delta f_D = 250 \) Hz and \( \Delta f_D = 167 \) Hz respectively, for different values of the initial phase. The values in the two figures are normalized with respect to the correlation maximum \( A_{IN}A_{LOC}^{-\frac{N}{2}} \).

Figure 8.5 shows the presence of small fluctuations due to the initial phase, which are, however, negligible with respect to the variations due to the not perfect alignment of the local Doppler with the incoming one. Figure 8.6 shows the same graphs, obtained for the 1–bit ADC scheme. Even in this case the variations in the maximum of the correlation peak due to the initial phase are negligible with respect to the variations due to the not perfect alignment of the local Doppler with the incoming Doppler.
8.2 – Search scheme analysis in the discrete Doppler domain

The analysis of the previous figures show that the variation of the envelope detector output with the initial phase and with the incoming Doppler shift decreases as the residual carrier increases and it is particularly harmful for frequencies in the range \([0 \div 2 \text{ kHz}]\), where the output value in the correct bin is more sensible to the initial phase. It is not convenient, therefore, that the residual carrier is lower than 2 kHz.

8.2.3 Envelope detector output in case of code delay alignment for variable input Doppler shift and for various integration times

The envelope detector output is also a function of the integration time. Figure 8.7 shows the correlation peak when the incoming signal Doppler varies on a frequency range of \([4100 \text{ kHz} \div 4400 \text{ kHz}]\) and the local search Doppler is set to \(f_D = 4.250 \text{ kHz}\) for one, two
and three code periods, corresponding to $T = 4$ ms, $T = 8$ ms and $T = 12$ ms. The Galileo signal has been considered with the floating–point and 1–bit ADC input system.

If the Doppler frequency step is $\Delta \hat{f}_D = 250$ Hz for the three integration times, the correlation peak decreases at the bin ends as the integration time increases. It goes from $0.65A_{IN}A_{LOC} N^2$ when the integration time is one code period to the zero value when the integration time is two code periods. In order to maintain the same value of the envelope detector output at the bin ends when the integration time increases, the Doppler search step must be reduced by a factor equal to the number of code periods used for the integration. This situation is depicted in Figure 8.8. The integration time goes from $T = 4$ ms to $T = 8$ ms to $T = 12$ ms and the Doppler search step is reduced from $\Delta \hat{f}_D = 250$ Hz to $\Delta \hat{f}_D = 125$ Hz to $\Delta \hat{f}_D = 84$ Hz.

The same results apply to the other implementations of the acquisition system and to
8.2 – Search scheme analysis in the discrete Doppler domain

Figure 8.5. Infinite-bit ADC: envelope detector output computed over one code period for the bins corresponding to $\hat{f}_D = 250$ Hz and $\hat{f}_D = 166.67$ Hz towards $f_D = 3.75$ kHz, increasing the Doppler frequency step of $\Delta \hat{f}_D = 250$ Hz and $\hat{f}_D = 166.67$ Hz respectively, in the case of the Galileo BOC(1,1) signal for various values of the phase.

Figure 8.6. 1 bit ADC: envelope detector output computed over one code period for the bins corresponding to $f_D = 250$ Hz and $f_D = 166.67$ Hz towards $f_D = 3.75$ kHz, increasing the Doppler frequency step of $\Delta f_D = 250$ Hz and $f_D = 166.67$ Hz respectively in the case of the Galileo BOC(1,1) signal for various values of the phase.

the other Doppler frequency steps.
Figure 8.7. Infinite-bit ADC: envelope detector output performed over different code periods for the bin corresponding to $\hat{f}_D = 4.250$ kHz when the input Doppler frequency varies in the range $[4.100 \text{ kHz} \div 4.400 \text{ kHz}]$, the Doppler search step is $\Delta \hat{f}_D = 250$ Hz and the initial phase is zero.

Figure 8.8. Infinite-bit ADC: envelope detector output performed over different code periods for the bin corresponding to $\hat{f}_D = 8.0$ kHz when the input Doppler frequency varies in the range $[4.100 \text{ kHz} \div 4.400 \text{ kHz}]$, the initial phase is zero and the Doppler search step is reduced of a factor equal to the number of code periods used for the integration.
8.3 Doppler estimate offset effect

As for the case of the correlation loss due to the code phase offset presented in section 7.3, the effect shown in the previous section can be interpreted as a loss of the correlation peak incurred by an arbitrary Doppler frequency distributed over the whole search space. In order to perform this analysis and formalize the simulation results of sections 8.2.2 and 8.2.3, the general expression of the envelope detector output is derived, using again the expression (C.1) and (C.2) stated in appendix C. Equations (8.4) and (8.6) can be written in the general form

\[ R_I = A_{IN}A_{LOC} \frac{N}{2} \left\{ D_N \left[ \pi(F_D + \hat{F}_D) \cos \left[ 2\pi(F_D + \hat{F}_D) \theta + \pi(F_D + \hat{F}_D)(N - 1) + \phi \right] \right] + D_N \left[ \pi(F_D - \hat{F}_D) \cos \left[ 2\pi(F_D - \hat{F}_D) \theta + \pi(F_D - \hat{F}_D)(N - 1) + \phi \right] \right] \right\} \] (8.12)

for the in-phase branch and

\[ R_Q = A_{IN}A_{LOC} \frac{N}{2} \left\{ D_N \left[ \pi(F_D + \hat{F}_D) \sin \left[ 2\pi(F_D + \hat{F}_D) \theta + \pi(F_D - \hat{F}_D)(N - 1) + \phi \right] \right] + D_N \left[ \pi(F_D - \hat{F}_D) \sin \left[ 2\pi(F_D - \hat{F}_D) \theta + \pi(F_D - \hat{F}_D)(N - 1) + \phi \right] \right] \right\} \] (8.13)

for the quadrature-branch.

Following the same considerations pointed out in section 7.3, comparing the expressions (8.12) and (8.13) with the expressions (8.8) and (8.9) and finally neglecting the effect of the terms \( F_D + \hat{F}_D \), which is practically meaningful, the amplitude correlation loss due to the Doppler estimate offset can be modeled as

\[ \beta_{loss} \simeq \left| D_N \left[ \pi(F_D - \hat{F}_D) \right] \right| \] (8.14)

This amplitude loss is depicted in Figure 8.9. The integration time goes from \( T = 4 \, ms \) to \( T = 8 \, ms \) to \( T = 12 \, ms \) and the Doppler search step is reduced from \( \Delta \hat{f}_D = 250 \, Hz \), \( \Delta \hat{f}_D = 125 \, Hz \) to \( \Delta \hat{f}_D = 84 \, Hz \) exactly as in the case of the simulation results of Figure 8.8. Comparing these two graphs, it is possible to evince the validity of Equation (8.14) to represent the influence of the Doppler, in terms of amplitude loss, of the envelope output.

Equation (8.14) can be written in the logarithmic form, according to the notation generally used for expressing the amplitude loss, as already done for the code delay loss

\[ \beta_{loss, dB} \simeq 20 \log_{10} \left| D_N \left[ \pi(F_D - \hat{F}_D) \right] \right| \]

\[ \simeq 20 \log_{10} \left| \frac{\sin \left[ \pi N (F_D - \hat{F}_D) \right]}{\sin \left[ \pi (F_D - \hat{F}_D) \right]} \right| \] (8.15)

A plot of this relationship is finally presented in Figure 8.10.
8.4 Working frequency and Doppler search steps

The considerations carried out in the previous sections provide a design criterion to select the best working frequency and Doppler steps for an acquisition system.

The minimum residual carrier should be high enough to minimize the variation of
the envelope detector output due to the initial carrier phase. Sections 8.2.1 and 8.2.2 have shown how a residual carrier greater than 2 kHz is already sufficient to neglect this influence. Considering that the maximum Doppler frequency shift for a low-speed user, according to the results of Section 4.4, is ±5 kHz the center baseband frequency should assume at least the value

\[ f_{BB} = 7.0 \text{ kHz} \]  

(8.16)

From the analysis of Section 8.2.1, the Doppler frequency steps that minimize the variation of the maximum of the envelope detector output in correspondence of the correct cell are \( \Delta f_D = 125 \text{ Hz} \) for the Galileo BOC(1,1) modulation, or multiples of those values, when the integration time is one code period. According to the results of Section 8.2.3, in order to make acceptable the variation of the envelope detector output due to the not correct alignment of the local Doppler with the incoming Doppler signal, the Doppler frequency step should be reduced of a factor equal to the number of code periods used in the Average and Dump block of the scheme of Figure 7.1.

In the receiver, increasing the number of code periods for the integration leads to a double increase in complexity. More time is required for the summation and closer frequency steps are needed to analyze the search space.
8 – Working frequency and required Doppler frequency steps
Chapter 9

Description of different acquisition schemes

The problem of signal acquisition is commonly encountered in all the CDMA applications. Many acquisition schemes for CDMA signal can be found in literatures. However, many of them are not suitable for navigation purposes where the signal–to–noise ratio is very low. Moreover, many authors, do not consider the residual carrier acquisition but only the code synchronization. This is the case of the articles in references [29], [30], [31] and [32].

In reference [15] a complete analysis of the main acquisition schemes suitable to be used in a Navigation receiver are analyzed and in this chapter this analysis will be recalled.

More in detail in this chapter the acquisition systems that deal with both code delay and Doppler frequency shift synchronization will be presented, which is the particular scenario a navigation receiver has to deal with and their system complexity pointed out.

9.1 Serial search scheme

The serial search scheme, which has already been presented in Chapter 7 is shown in Figure 9.1. It is the implementation proposed in references [4], [13], [33] and [34].

The Serial Search functioning principles have already been presented in Chapter 7 and the only remark which has to be add is that the correlation between the received and local codes is generally performed over one or more entire code periods.

It is useful to evaluate the acquisition scheme complexity in order to compare the various strategies. A good index of the system complexity is the number of multiplications required by the entire process. Naming $N_c$ the number of code delays to search for and $N_p$ the number of code periods used for the average and dump process, the total number of samples used in a single integration is $N = N_c N_p$. Therefore, neglecting the calculation
of the envelope and all the summations, the number of multiplications \( M \) required for the whole search space scanning, with \( N_D \) Doppler bins and \( K \) non–coherent integration periods, is

\[
M = K N_D \cdot (N + 2N) = 3K N_D \cdot N
\]  

(9.1)

This 1–bit implementation reduces the complexity of the RF front–end, since no AGC is needed in order to set the correct input signal level for the ADC, and the first multiplication of the incoming signal with the replica code is very easy to implement. As addressed in [15], it can be performed through a two–by–two matrix, so it can be neglected in the complexity computation. The number of multiplications required for the scanning of the entire matrix is, therefore

\[
M = K N_D \cdot 2N
\]

(9.2)

Even if the current technology allows to use an ADC able to represent the signal with 8 or more bits, in this thesis the case of a solution based on a single 1-bit ADC will be taken into account as a possible solution for a very simple receiver implementation.

9.2 Parallel acquisition in time delay domain: Fast Acquisition scheme

The scheme shown in Figure 9.2 performs a parallel acquisition in time delay domain. This system is described in references [14], [35], [36] and [37] and it is often addressed with the name of Fast Acquisition Scheme.

The input digital signal \( x[n] \) is split in the in–phase and quadrature–phase branches. The resulting signals become the real part, \( x_{\text{Re}}[n] \), and the imaginary part, \( x_{\text{Im}}[n] \), of the FFT input. The complex samples obtained from the FFT operation are then multiplied by
the complex conjugates samples of the local code $X_{\text{LOC},s}[n]$ and then FFT inverse transformed. The described operations perform a circular correlation, as described in Appendix B), and provides the complete correlation function over the integration period. In this way, a whole row of the search matrix of Figure 5.2, corresponding to all the possible code delays, is computed.

The number of samples of the FFT is equal to the number of the code steps in the search space. As for the serial search scheme two possible ways to increase the robustness to the noise level are possible: the first solution is to compute the FFT over more than one code period, the second solution uses a non-coherent summation of more envelope detector inputs. With the second strategy, however, lower performance can be achieved, since it must be taken into account the loss introduced by the squaring operation.

This acquisition system is theoretically faster than the serial search scheme, since only $N_D$ Doppler frequency steps are needed and the $N_c$ code delay steps are computed in parallel, but this gain is obtained only if the signal processor is fast enough to compute a $N_c$ point FFT within one dwell time.

The complexity of this acquisition system can be computed, discarding the needed operations to calculate the envelope, as

$$M = KN_D(2N + 3N \ln N + N) = KN_DN(3 \ln N + 3)$$

(9.3)

where the FFT complexity has been considered equal to $N \ln N$.

This solution can be considered an alternative implementation to the serial search scheme, that produces the same results just performing the correlation by means of FFTs.
### 9.3 Parallel acquisition in Doppler frequency domain

The acquisition system which performs the parallel search in the Doppler frequency domain is shown in Figure 9.3. The digital input signal is multiplied with the local PRN replica code delayed of the local estimate \( \hat{\theta} \) and the FFT of the obtained burst of samples is computed. The result is passed through an envelope detector and all the desired frequency step bins are investigated in parallel.

![Figure 9.3. Parallel acquisition in Doppler frequency domain: FFT in Doppler frequency domain](image)

The operation performed by this scheme is

\[
X(nf_0) = \frac{T}{N} \sum_{n=0}^{N-1} \pi(mT_0) e^{-jn \frac{2\pi}{N} m} \quad \text{for} \ n = 0, \ldots, N - 1
\]  

(9.4)

where \( X(nf_0) \) are the transformed data, \( \pi(mT_0) = A_{IN} A_{LOC} \pi(mT_0) e^{j(mT_0 + \hat{\theta})} \) are the time domain data, \( N \) is the total number of samples, \( T \) is the time duration of the input array, \( T_0 \) is the time spacing between the input samples and \( f_0 = \frac{1}{T} \) is the FFT frequency resolution. The operation so performed, taking into account that \( e^{jx} = \cos(x) + j\sin(x) \) and the orthogonality between sinusoids, can be shown to be identical to the operations made by the serial or fast acquisition scheme.

For the acquisition of a signal the FFT has to be performed on complex samples and produces complex numbers and it is therefore necessary to compute the absolute value of each output sample in order to be able to compare it to a threshold.

The number of Doppler frequency steps and FFT points are determined by the number of code periods used for the FFT calculation. If \( T \) is the temporal duration of the input samples of the FFT, the frequency resolution is

\[
\Delta f_D = \frac{1}{T}
\]  

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9.3 – Parallel acquisition in Doppler frequency domain

and where the number of FFT samples, as for the case of the fast acquisition scheme, depends on the sampling rate and the coherent integration time.

In order to increase the signal–to–noise ratio at the envelope detector output, the same solutions proposed for the serial search and fast acquisition schemes are possible. Increasing the number of code periods over which the FFT is computed, that is equivalent to using more code periods in the summation of the serial search scheme, and computing the summation of $K$ envelope detector output of different FFT stages, as proposed in reference [35]. The differences between the two cases, as it will be shown by simulation results, is the same of the two solutions for the parallel search in the time delay domain.

This acquisition system requires, for each Doppler row of the serial search space, only one FFT calculation, but the whole time delay domain has to be scanned serially. The number of required multiplications for the entire matrix analysis, discarding the operation of the envelope detector, are

$$M = K(N + N \ln N) = KN(1 + \ln N)$$ (9.5)
9 – Description of different acquisition schemes
Chapter 10

Acquisition schemes theoretical analysis

In this chapter a detailed theoretical analysis based on the classical serial search scheme will be carried out. The theoretical investigation of the probability density functions involved in the acquisition process for the floating-point and 1-bit input will be investigated. In particular, the probability density functions of the aligned and non-aligned case will be determined.

10.1 Detection criteria

The generic acquisition technique determines for each satellite the search space, which can be scanned in a serial or in a parallel way. A threshold is set on the basis of the required false alarm probability and the signal is declared present when a cell value overcome it. This is the common acquisition methodology as reported in references [4] and [13].

It is here remarked that all the acquisition schemes presented in Chapter 9 are equivalent, the only difference is the way how the search space is determined, so the following definitions of false alarm and detection probabilities apply to all of them.

In order to carry out the analysis, for the sake of simplicity, the serial search scheme of Figure 10.1 will be taken as reference. Considering the scheme of Figure 7.1, the envelope detector output can be considered a random process characterized by a density function depending on the fact that the considered cell in the search space corresponds to the right or wrong signal alignment.

Naming $f_{na}(x)$ the probability density function of the process at the envelope detector output under the hypothesis of signal misalignment, the false alarm probability is defined
as the probability that the signal is declared present in a wrong cell:

\[ P_{fa} = \int_{V_t}^{+\infty} f_{na}(x) \, dx \]  

(10.1)

where \( V_t \) is the acquisition system threshold. The threshold sets the desired false alarm probability and it can be obtained by inverting the previous equation.

In the same way, naming \( f_a(x) \) the probability density function of the envelope detector output, it is possible to define the detection probability as the probability that the signal is detected under the condition of perfect code delay and Doppler shift alignment

\[ P_d = \int_{V_t}^{+\infty} f_a(x) \, dx \]  

(10.2)

The missed detection probability is defined as the complement of the detection probability

\[ P_{miss} = \int_{0}^{V_t} f_a(x) \, dx = 1 - P_d \]  

(10.3)

### 10.2 Probability density functions for the floating–point input

With the notation introduced in Chapter 7 and with reference to Figure 9.1, the received digital signal can be written in the form

\[ x[n] = x_C[n] + n_w[n] = A_{IN} x_{IN}[n + \theta] \cos [2\pi F_D(n + \theta) + \phi_1] + n_w[n] \]  

(10.4)

where \( F_D \) includes both the residual center frequency and the incoming Doppler frequency shift. The signals at the in–phase and quadrature–phase branches of the correlator, after the multiplication with the local code and the local sine and cosine, are
10.2 – Probability density functions for the floating-point input

\[ x_1[n] = \{ A_{\text{IN}}x_{\text{IN}}[n + \theta] \cos [2\pi F_D(n + \theta) + \phi_1] + n_w[n] \} \cdot \]
\[ A_{\text{LOC}}x_{\text{LOC}}[n + \hat{\theta}] \cos [2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2] \] (10.5)

\[ x_Q[n] = \{ A_{\text{IN}}x_{\text{IN}}[n + \theta] \cos [2\pi F_D(n + \theta) + \phi_1] + n_w[n] \} \cdot \]
\[ A_{\text{LOC}}x_{\text{LOC}}[n + \hat{\theta}] \sin [2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2] \] (10.6)

The correlator outputs can be written as

\[ R_I(\theta, \hat{\theta}, F_D, \hat{F}_D) = \sum_{n=0}^{N-1} \{ A_{\text{IN}}x_{\text{IN}}[n + \theta] \cos [2\pi F_D(n + \theta) + \phi_1] \cdot \]
\[ A_{\text{LOC}}x_{\text{LOC}}[n + \hat{\theta}] \cos [2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2] \}
\[ + \sum_{n=0}^{N-1} A_{\text{LOC}}x_{\text{LOC}}[n + \hat{\theta}]n_w[n] \cos [2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2] \] (10.7)

\[ R_Q(\theta, \hat{\theta}, F_D, \hat{F}_D) = \sum_{n=0}^{N-1} \{ A_{\text{IN}}x_{\text{IN}}[n + \theta] \cos [2\pi F_D(n + \theta) + \phi_1] \cdot \]
\[ A_{\text{LOC}}x_{\text{LOC}}[n + \hat{\theta}] \sin [2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2] \}
\[ + \sum_{n=0}^{N-1} A_{\text{LOC}}x_{\text{LOC}}[n + \hat{\theta}]n_w[n] \sin [2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2] \] (10.8)

10.2.1 Probability density function for the non–aligned case

If the local generated code is not aligned with the received signal, the product \(x_{\text{IN}}[n + \theta]x_{\text{LOC}}[n + \hat{\theta}] \approx 0\), then the in–phase branch of the correlator expressed by Equation (10.7) becomes

\[ R_I = \sum_{n=0}^{N-1} A_{\text{LOC}}x_{\text{LOC}}[n + \hat{\theta}]n_w[n] \cos [2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2] \] (10.9)

which is a random gaussian process with zero mean

\[ \mu_{R_I} = 0 \] (10.10)
and variance

$$\sigma^2_{R_i} = A^2_{LOC}\sigma^2_n \sum_{n=0}^{N-1} \cos^2 \left[ 2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2 \right] \simeq A^2_{LOC} \frac{N}{2} \sigma^2_n$$  \hspace{1cm} (10.11)

The same applies for the quadrature–phase and the resulting random process has zero mean

$$\mu_{R_Q} = 0$$  \hspace{1cm} (10.12)

and variance

$$\sigma^2_{R_Q} = A^2_{LOC}\sigma^2_n \sum_{n=0}^{N-1} \sin^2 \left[ 2\pi \hat{F}_D(n + \hat{\theta}) + \phi_2 \right] \simeq A^2_{LOC} \frac{N}{2} \sigma^2_n$$  \hspace{1cm} (10.13)

The approximation for the variance is true for large residual carrier\(^1\) and it will be justified in Section 10.2.4.

The probability density function for the non–aligned case can be derived directly (see [38]) as

$$f_{na_R}(r) = \frac{r}{A^2_{LOC}(N/2)\sigma^2_n} e^{-\frac{r^2}{A^2_{LOC}N\sigma^2}} u(r)$$  \hspace{1cm} (10.14)

which is known under the name of Rayleigh distribution and where \(u(r)\) is the unitary echelon function.

In order to improve the correlation performances the detection and the decision can be taken on the summation on several samples of the correlator output before the envelope operation. In this case, according to the scheme of Figure 10.1 the envelope input \(G\) can be written as

$$G = \sum_{i=1}^{K} R_i^2[i] + \sum_{i=1}^{K} R_Q^2[i]$$  \hspace{1cm} (10.15)

The summation on \(K\) independent instances of \(R_i^2[i]\) and \(R_Q^2[i]\) produce a mean effect that reduces the noise impact. From the consideration addressed in Appendix D, the \(R_i^2[i]\) and \(R_Q^2[i]\) are distributed according to a \(\Gamma\) distribution

$$R_i^2[i] \sim \Gamma\left(\frac{1}{2}, 2\sigma^2\right)$$  \hspace{1cm} (10.16)

$$R_Q^2[i] \sim \Gamma\left(\frac{1}{2}, 2\sigma^2\right)$$  \hspace{1cm} (10.17)

\(^1\)The approximation is already valid when the residual carrier assumes the value of few Kilohertz.
with $\sigma^2 = A_{\text{LOC}}^2 \frac{N}{2} \sigma_n^2$. Using again the property reported in Appendix D, it is easy to show that

$$G \sim \Gamma(K, 2\sigma^2)$$

(10.18)

where $K$ is the number of terms involved in the averaging of the correlation function. In this way the expression of the probability density function of the variable $G$ can be expressed as follows

$$f^k_G(x) = \frac{1}{\Gamma(K)(2\sigma^2)^K} x^{K-1} e^{-\frac{x}{\sigma^2}} u(x)$$

$$= \frac{1}{2^K (K - 1)! \sigma^2} x^{K-1} e^{-\frac{x}{\sigma^2}} u(x)$$

(10.19)

As last remark, Equation (10.19) is the general expression for the distribution of the statistic prior of the envelope detector output, in fact Equation (10.14) can be easily derived from (10.19) considering the case $K = 1$.

### 10.2.2 Probability density function for the aligned case

When the Doppler frequency shift is $F_D = \hat{F}_D = \frac{k}{2N}$ and considering a situation of perfect signals alignment, Equation (10.7) becomes

$$R_I = \frac{A_{\text{IN}} A_{\text{LOC}}}{2} N \cos \phi + \sum_{n=0}^{N-1} A_{\text{LOC}} x_{\text{LOC}[n+\hat{\theta}]} [n] \cos \left[ 2\pi \hat{F}_D (n + \hat{\theta}) + \phi_2 \right]$$

(10.20)

where $\phi$ denotes the difference between phase difference between the received signal and the local oscillator. This is a gaussian random process with mean value equal to

$$\mu_{R_I} = \frac{A_{\text{IN}} A_{\text{LOC}}}{2} N \cos \phi$$

(10.21)

and variance

$$\sigma_{R_I}^2 = A_{\text{LOC}}^2 \sigma_n^2 \sum_{n=0}^{N-1} \cos^2 \left[ 2\pi \hat{F}_D (n + \hat{\theta}) + \phi_2 \right] \simeq A_{\text{LOC}}^2 \frac{N}{2} \sigma_n^2$$

(10.22)

Under the previous conditions, it is possible to derive that the signal at the quadrature-phase branch of the envelope detector input is a gaussian random process with mean equal to

$$\mu_{R_Q} = -\frac{A_{\text{IN}} A_{\text{LOC}}}{2} N \sin \phi$$

(10.23)
and variance
\[ \sigma_{R_Q}^2 = A_{\text{LOC}}^2 \sigma_n^2 \sum_{n=0}^{N-1} \sin^2 \left[ 2\pi F_D (n + \hat{\theta}) + \phi_2 \right] \approx A_{\text{LOC}}^2 \frac{N}{2} \sigma_n^2 \] (10.24)

according to the consideration which will be explained in Section 10.2.4.

The probability density function of the envelope detector output can be derived studying the probability density function of the random process \( R = \sqrt{R_I^2 + R_Q^2} \).

The random processes at the envelope detector inputs are normally distributed and independent, then it is possible to write the pdf of the envelope detector output conditioned to the random initial phase \( \Phi \) (see [38]) as
\[
    f_{\Phi \mid R}(r \mid \phi) = \frac{r}{2\pi \sigma^2} e^{-\frac{r^2 + \alpha^2}{2\sigma^2}} \int_{0}^{2\pi} e^{\frac{r \alpha \sigma^2 \cos(\phi - \theta)}{2\sigma^2}} d\theta
\]

where in order to keep the notation more compact the following variable has been introduced: \( \alpha = A_{\text{IN}} A_{\text{LOC}} \frac{N}{2} \) and \( \sigma^2 = A_{\text{LOC}}^2 \frac{N}{2} \sigma_n^2 \). After some manipulations the previous expression becomes
\[
    f_{\Phi \mid R}(r \mid \phi) = \frac{r}{2\pi \sigma^2} e^{-\frac{r^2 + \alpha^2}{2\sigma^2}} \int_{0}^{2\pi} e^{\frac{r \alpha \sigma^2 \cos(\phi - \theta)}{2\sigma^2}} d\theta
\]

\( R \) and \( \Phi \) are independent random variables, so that the desired pdf can be calculated as
\[
    f_{\Phi}(r) = \int_{-\infty}^{+\infty} f_{\Phi \mid R}(r \mid \phi) f_{\Phi}(\phi) d\phi
\]

The random variable \( \Phi \) is uniformly distributed over the interval \([-\pi, \pi]\), hence, with the change of variable \( \alpha = \phi - \theta \),
\[
    f_{\Phi}(r) = \frac{r}{2\pi \sigma^2} e^{-\frac{r^2 + \alpha^2}{2\sigma^2}} \int_{0}^{2\pi} e^{\frac{r \alpha \sigma^2 \cos(\phi - \theta)}{2\sigma^2}} d\theta
\]

The pdf of the envelope detector output for the aligned case can be derived by considering the modified Bessel function of zero order, which has the expression
\[
    I_0(x) = \frac{1}{2\pi} \int_{0}^{2\pi} e^{x \cos(\theta)} d\theta
\]
and noticing that \( e^{x \cos(\theta)} \) is periodic with period \( 2\pi \), so that the same result is obtained if the exponential is integrated over \([0, 2\pi]\) or \([-\theta, 2\pi - \theta]\). It is then possible to write
\[
    \frac{1}{2\pi} \int_{-\theta}^{2\pi - \theta} e^{\frac{r \alpha \sigma^2 \cos(\phi)}{2\sigma^2}} d\alpha = I_0 \left( \frac{r \alpha}{\sigma^2} \right)
\]

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The probability density function in the aligned case assumes the final expression

\[ f_{aR}(r) = \frac{r}{\sigma^2} e^{-\left(\frac{r^2 + \alpha^2}{2\sigma^2}\right)} I_0 \left( \frac{r\alpha}{\sigma^2} \right) \]  \hspace{1cm} (10.25)

In Appendix E the general expression for the probability density function, when \( K \) samples of the correlator output are summed up to improve the noise reduction performance, is derived

\[ f_{kR}(r) = \frac{\sqrt{k\alpha}}{\sigma^2} \left( \frac{r}{\sqrt{k\alpha}} \right)^k e^{-\frac{1}{2} \left( \frac{r^2 + k\alpha^2}{\sigma^2} \right)} I_{k-1} \left( r\sqrt{k\alpha} \right) \]  \hspace{1cm} (10.26)

10.2.3 Probability density function

The probability density functions for the aligned and non-aligned case are depicted in Figure 10.2 for the carrier-to-noise ratios \( \text{CNo} = 30 \text{ dB–Hz} \) and \( \text{CNo} = 35 \text{ dB–Hz} \) respectively. The Figure shows the case of integration over one code period.

![Probability density functions](image)

Figure 10.2. Probability density functions for the aligned and non-aligned cases for \( \text{CNo} = 30 \text{ dB–Hz} \) and \( \text{CNo} = 35 \text{ dB–Hz} \) respectively and integration performed over one code period

When the CNo is not very high the threshold setting is more difficult. In fact, in order to have a high detection probability the threshold should be small, but this would leads to non negligible false alarm probability.

Figure 10.3 shows the histograms of the envelope detector output in the aligned and non-aligned case obtained by means of computer simulations. A \( \text{CNo} = 30 \text{ dB–Hz} \) is considered and 100000 samples used to estimate the pdfs. The histogram bins well fit the theoretical probability density functions, which proves that the validity of the previous investigation.
Figure 10.3. Histogram and probability density function for the envelope detector output in the aligned and non-aligned case for $\text{CNo} = 30\text{ dB–Hz}$ and integration performed over one code period.

The same situation, but for $\text{CNo} = 35\text{ dB–Hz}$, is shown in Figure 10.4.

Figure 10.4. Histogram and probability density function for the envelope detector output in the aligned and non-aligned case for $\text{CNo} = 35\text{ dB–Hz}$ and integration performed over one code period.
10.2.4 Random process variance analysis

The variance of the random process derived in the previous sections do not depend from the signal, so the variance expression is identical both in the aligned and non-aligned case. They have been approximated to the value $A_{\text{LOC}}^2 \frac{N}{2} \sigma_n^2$, but in the general case, the variance expressions are not equal and the probability density functions are no more the ones of Equations (10.25) and (10.14).

The exact expressions of the process variance are

$$\sigma_{RI}^2 = A_{\text{LOC}}^2 \sigma_n^2 \sum_{n=0}^{N-1} \cos^2 \left[ 2\pi \hat{F}_D (n + \hat{\theta}) \right]$$

$$\sigma_{RQ}^2 = A_{\text{LOC}}^2 \sigma_n^2 \sum_{n=0}^{N-1} \sin^2 \left[ 2\pi \hat{F}_D (n + \hat{\theta}) \right]$$

where, for the sake of simplicity, the dependence on the initial phase $\phi$ has been neglected. After some manipulation, they can be written in the form

$$\sigma_{RI}^2 = \frac{A_{\text{LOC}}^2 \sigma_n^2}{2} \left\{ N + \sum_{n=0}^{N-1} \cos \left[ 4\pi \hat{F}_D (n + \hat{\theta}) \right] \right\}$$

$$\sigma_{RQ}^2 = \frac{A_{\text{LOC}}^2 \sigma_n^2}{2} \left\{ N - \sum_{n=0}^{N-1} \cos \left[ 4\pi \hat{F}_D (n + \hat{\theta}) \right] \right\}$$

and with the consideration derived in Appendix C, they assume the expression

$$\sigma_{RI}^2 = \frac{A_{\text{LOC}}^2 \sigma_n^2}{2} N \left\{ 1 + D_N \left( 2\pi \hat{F}_D \right) \cos \left[ 4\pi \hat{F}_D \hat{\theta} + 2\pi \hat{F}_D (N - 1) \right] \right\} \quad (10.27)$$

and

$$\sigma_{RQ}^2 = \frac{A_{\text{LOC}}^2 \sigma_n^2}{2} N \left\{ 1 - D_N \left( 2\pi \hat{F}_D \right) \cos \left[ 4\pi \hat{F}_D \hat{\theta} + 2\pi \hat{F}_D (N - 1) \right] \right\} \quad (10.28)$$

The maximum difference of the quantity $|\sigma_I^2 - \sigma_Q^2|$ as a function of the local Doppler frequency shift is simply

$$\left| \sigma_I^2 - \sigma_Q^2 \right| = A_{\text{LOC}}^2 \sigma_n^2 N \left| D_N \left( 2\pi \hat{F}_D \right) \cos \left[ 4\pi \hat{F}_D \hat{\theta} + 2\pi \hat{F}_D (N - 1) \right] \right| \quad (10.29)$$

Figure 10.5 shows the quantity $|\sigma_I^2 - \sigma_Q^2|$ as a function of the local Doppler frequency shift. It is normalized with respect to the asymptotic value $A_{\text{LOC}}^2 \sigma_n^2 N/2$.

For low values of the residual carrier $\hat{F}_D$, the difference between the absolute values of the variances over the two branches is even the double of the asymptotic value, which is $A_{\text{LOC}}^2 \sigma_n^2 N$, while, as the residual carrier increases, the difference reduces. Figure 10.5 shows that for residual carrier frequencies high enough, id est $f_{BB} > 1$ kHz, the difference between the two variances can be neglected.
10.3 Probability density functions for the 1-bit ADC

The analysis of the probability density functions for the aligned and non-aligned case for the 1-bit input signal is extremely complex and it will not be carried out in thesis. Reference [15] provides more detail on this specific case. In particular, by means of computer simulations it is shown that, both in the aligned and non aligned case, the random process can be assumed gaussian. Moreover, for the non-aligned case the random process at the envelope detector input are zero mean gaussian processes with variance

\[ \sigma^2_{RI} = \sigma^2_{RQ} = \frac{N^2}{2} A^2_{LOC} \]  

and then, the probability density function of the envelope detector output for the non-aligned case, from reference [38], assumes the expression

\[ f_{\text{naR}}(r) = r \frac{r}{A^2_{LOC}N/2} e^{-\frac{r^2}{A^2_{LOC}N}} \quad \text{for} \quad r \geq 0 \]  

10.4 False Alarm Probability

Different techniques can be found in literature for the threshold setting. References [39], [40] and [41] analyze some algorithms for the threshold setting in order to control both
the false alarm and the detection probability in CDMA acquisition systems. In these three articles, however, the considered signal-to-noise ratios are higher than the GPS and Galileo signal SNRs.

The threshold setting method that will be considered in the thesis stems from the one proposed in references [4], [13] and [33]. A value for the false alarm probability is chosen and the corresponding threshold is set on the basis of the probability density functions determined in the previous sections. Such threshold is an "optimum" threshold value, in the sense that it is the value that allows to obtain a specific false alarm probability.

The false alarm probability is equal to

\[ P_{fa}(V' t) = \int_{V' t}^{+\infty} f^k(x)dx \]

where \( V' t \) is the square of the searched threshold since \( f^k(x) \) is the statistic before the envelope operation. Hence, using Equation (10.19), it is easy to obtain

\[ P_{fa}(V' t) = \frac{1}{2K(K-1)!\sigma^{2K}} \int_{V' t}^{+\infty} x^{K-1}e^{-\frac{x^2}{2\sigma^2}}dx \]  

(10.32)

The final expression can be then obtained integrating by part \( K-1 \) times. After some algebraic manipulations it is possible to write the relationship between the probability of false alarm and the threshold

\[ P_{fa}(V' t) = \frac{1}{2K(K-1)!\sigma^{2K}} \sum_{i=1}^{K} \frac{(2\sigma^2)^i(K-1)!}{(K-i)!} V'^{K-i}t^{-i}e^{-\frac{V'^2}{2\sigma^2}} \]  

(10.33)

\[ = e^{-\frac{V'^2}{2\sigma^2}} \sum_{i=0}^{K-1} \frac{1}{i!} \left( \frac{V'^2}{2\sigma^2} \right)^i \]  

(10.34)

\[ = e^{-\frac{V'^2}{2\sigma^2}} \sum_{i=0}^{K-1} \frac{1}{i!} \left( \frac{V'^2}{2\sigma^2} \right)^i \]  

(10.35)

which has to be computed numerically. Finally the threshold at the output of the envelope detector is simply the square root of \( V' t \), i.e., \( V_t = \sqrt{V' t} \).

From Equations (10.11) and (10.13), it can be noticed that the threshold for the first implementation of the acquisition system depends on the noise variance, besides the number of samples used in the integration. This means that an acquisition system of this kind has to evaluate the noise variance before performing the acquisition. In reference [33] two methods for the estimation of the noise variance are proposed. According to the first one the input signal is correlated with an unused PRN code and a reference noise is synthesized, from which the input noise variance can be computed. In this case a second correlator is needed and an unused PRN code has to be generated. The second
method is less preferable. The same input signal is used for the noise variance estimation, by delaying it of one integration time. If the integration time is extended over more than one code period, the estimate of the input noise variance has to be performed over the same number of samples used in the integration, with an increase in the acquisition time.

Sections 10.2.1 and 10.3 have shown how for the non-aligned case the implementation of the acquisition scheme with infinite or a single bit ADC are practically identical, but for the correlation variance \( \sigma^2 \), which is noise independent for the 1-bit ADC implementation. Equation (10.35) can be then used in the threshold settings, both for the infinite and the 1-bit ADC scheme, according with the following rule

- Floating point ADC implementation \( \rightarrow \sigma^2 = A^2_{\text{LOC}} \sigma^2_{N} \frac{N}{2} \)
- 1-bit ADC \( \rightarrow \sigma^2 = A^2_{\text{LOC}} \frac{N}{2} \)

10.5 Detection Probability

In the previous sections the detection law for an acquisition system is derived supposing the system to be able to perfectly recover the code delay and the Doppler frequency shift. However, in real applications, these conditions are rarely verified. Neither the code delay nor the Doppler shift are exactly in the set of delays and frequencies used of the search space resolution. This condition is the causes of an additional impairment, or loss, which reduces the amplitude of the correlation peak used in the signal acquisition. Section 7.3 deals with the correlation loss due to the code phase offset effect, while Section 8.3 shows the impairment of a Doppler misalignment. These two effects have to be taken into account in order to evaluate the correct detection probability of a real system.

The signal level depends on two main non-idealities: in other words

- The code loss due to an arbitrary code phase falling in between the correlation resolution, which has been demonstrated in Section 7.3 to be

  \[ \alpha_{\text{loss}} = |R_{\text{BOC}}(\tau)| \]

- The loss incurred by an arbitrary Doppler frequency falling in between the frequency bins, which has been shown in Section 8.3 to be well modeled with

  \[ \beta_{\text{loss}} \simeq |D_{N} \left\{ \pi(F_{D} - \hat{F}_{D}) \right\}| \]

Since these two effects are independent, remembering the consideration addressed in Sections 7.3 and 8.3 and neglecting the initial phase effect \( \phi \), the following approximation for the correlation output can take place

\[ 104 \]
\[ R_I \left( \theta, \hat{\theta}, F_D, \hat{F}_D \right) \simeq R_{BOC} \left( \theta - \hat{\theta} \right) D_N \left[ \pi (F_D - \hat{F}_D) \right] \\
\cdot \cos \left[ 2\pi (F_D - \hat{F}_D)\hat{\theta} + \pi (F_D - \hat{F}_D)(N - 1) \right] \tag{10.36} \]
\[ R_Q \left( \theta, \hat{\theta}, F_D, \hat{F}_D \right) \simeq R_{BOC} \left( \theta - \hat{\theta} \right) D_N \left[ \pi (F_D - \hat{F}_D) \right] \\
\cdot \sin \left[ 2\pi (F_D - \hat{F}_D)\hat{\theta} + \pi (F_D - \hat{F}_D)(N - 1) \right] \tag{10.37} \]

In order to consider the different losses correctly, the model of the distribution for the code phase offset and Doppler shift have to be modeled. The resolution used in the acquisition phase is usually of half chip/slot and therefore the maximum absolute phase offset \( \Delta \Theta \) can be assumed uniformly distributed between \( \pm \frac{1}{4} \) chip/slot, which can be written as follows

\[ P_{\Delta \Theta} (\theta) = \begin{cases} 
2, & \text{if } |\theta| \leq \frac{1}{4} \\
0, & \text{elsewhere} 
\end{cases} \tag{10.38} \]

In the same way, the Doppler frequency \( \Delta F_D \) can be assumed to be uniformly distributed between zero and half the maximum absolute frequency bin width

\[ P_{\Delta F_D} (f) = \begin{cases} 
N, & \text{if } |f| \leq \frac{1}{2N} \\
0, & \text{elsewhere} \end{cases} \tag{10.39} \]

The combined loss due to the two independent effects is the sum of the contribution of the two losses. Thus, according to the definition and the results derived in Appendix E, the detection probability including the code phase offset and Doppler frequency shift loss effect is

\[ P_d = 2N \int_{-\frac{1}{4}}^{\frac{1}{4}} \int_{-\frac{1}{2N}}^{\frac{1}{2N}} Q_k \left( \frac{\sqrt{k}}{\sigma} D_N (\pi f) R_{BOC} (\theta), \frac{\sqrt{V_i}}{\sigma} \right) df d\theta \tag{10.40} \]
Chapter 11

Acquisition of the Galileo BOC(1,1) modulation

The Galileo BOC(1,1) modulation on the L1 carrier is rather similar to the GPS C/A code, so that an acquisition system for Galileo signals can be designed on the basis of a GPS acquisition system. The differences and the similarities between the two architectures will be analyzed in this chapter and the GPS acquisition system, opportunely adapted, will be applied to the Galileo signal.

The standard acquisition technique, which detects the highest peak of the autocorrelation function, will be considered and its performances will be analyzed in detail.

In this chapter some algorithms which consider the particular shape of the Galileo BOC(1,1) autocorrelation function will also be considered. The algorithms specifically designed for Galileo signals will prove particularly successful against the detection problem of the Galileo BOC(1,1) correlation side lobes.

11.1 Acquisition systems for Galileo BOC(1,1) signal and comparison with the GPS system

In the thesis the Galileo BOC(1,1) modulation on the L1 carrier is considered. This signal is rather similar to the GPS C/A signal, as pointed out in Chapter 3, which can be derived from the GPS C/A code by applying to it a Manchester like coding.

One difference between the GPS and Galileo signals is their bandwidth, since the application of the Manchester coding on the C/A code causes a splitting of the code power spectral density. The single–sided bandwidth, that for the GPS C/A code is $B_s = 1.023 \text{ MHz}$, for the Galileo BOC(1,1) signal becomes $B_s = 2.046 \text{ MHz}$, as depicted in Figure 3.5. This means that, in order to process the Galileo signal in the same way as the GPS C/A signal, the ADC antialiasing filter of Figure 4.2 must have a single–sided
bandwidth twice larger than the corresponding GPS ADC anti-aliasing filter bandwidth. Therefore, the sampling frequency must be at least twice the GPS C/A code sampling frequency.

The sampling frequency value can be derived also considering that the Galileo BOC(1,1) slot rate is twice the GPS C/A chipping rate. The final result is that every slot of the Galileo BOC(1,1) modulation has a rate twice than the chipping rate of the GPS C/A code signal, i.e. $R_{BOC} = 2.046 \text{ Mslot/s}$. This implies that, in order to obtain about two samples per BOC(1,1) slot it is necessary to sample the Galileo signal at a rate that is twice greater than the sampling rate of the GPS C/A code.

From the previous considerations it follows that a Galileo receiver can be designed on the basis of a GPS receiver, but the number of samples to process, besides, is accordingly greater than the number of samples processed by the GPS receiver in the same condition of integration time. The global complexity of a Galileo BOC(1,1) receiver is, therefore, slightly increased with respect to the GPS C/A receiver, but the same applies to the acquisition performances, as it will be pointed out in the next sections, along with the positioning precision.

### 11.2 Receiver operative characteristics

The simulations performed to determine and analyze the acquisition systems and their optimum parameters aim to obtain the so-called Receiver Operative Characteristic, which will be named with the acronym ROC. This is the graph of the detection probability versus the false alarm probability, or, equivalently, of the missed detection probability versus the false alarm probability.

Both the simulated detection and the false alarm probabilities are obtained by means of the error-counting technique also known as Montecarlo technique. A number of samples is fed to the acquisition system and the detection and false alarm probabilities are determined as the ratio between the total number of trials over the number of signal detections or false alarms incurred in the simulation.

The required number of trials depends on the probability which is to estimate. An estimation is generally considered sufficiently reliable if at least 100 “events” are detected. In this particular case this means that at least 100 false detections have to be detected for the false alarm probability and 100 missed detections have to be counted for detection probability. In fact in normal situation the detection probability is reasonable higher that 0.5, so the missed detection probability, which is its complementary, must be considered for the simulation reliability.
11.3 Simulation parameters

The simulation parameters are based on the analysis of Chapters 7 and 8. The input signal is a baseband signal whose center frequency is

$$f_{BB} = 8 \text{ kHz}$$  \hspace{1cm} (11.1)

The sampling rate is set on the basis of the discussion of Chapters 6 and 7 and it is equal to

$$f_s = 2.0578 \times 2 \times R_{BOC} = 2.0578 \times 2 \times 1.023 \text{ MHz} = 4.210256 \text{ MHz}$$  \hspace{1cm} (11.2)

In the sampling process, the starting sampling point is a random variable which assumes values between $$[-T_c/4; T_c/4]$$, where $$T_c$$ is the slot period.

The initial phases of the received cosine and of the receiver local oscillator are random phases uniformly distributed between

$$\phi_{1,2} \in [-\pi; \pi]$$  \hspace{1cm} (11.3)

The Doppler frequency shift is in the range

$$f_D \in [-5 \text{ kHz}; 5 \text{ kHz}]$$  \hspace{1cm} (11.4)

and a random factor is used for the input signal frequency, so that it is not exactly equal to a frequency value of the searching grid. The variation of the system output with the variation of the incoming Doppler shown in Section 8.2.2 is, therefore, taken into account.

The number of Doppler frequency steps used in the acquisition system depends on the number of the Coherent integration time. The Doppler frequency step is equal to $$\Delta f_D = 250/N_p \text{ Hz}$$, as determined in Section 8.2.3.

The simulations are performed generally for the input carrier-to-noise ratio of $$CNo = 33 \text{ dB–Hz}$$, which is considered a realistic operational value for the Galileo BOC(1,1) modulation.

11.4 Single period integration time analysis and comparison

Parallel acquisition in time domain schemes, based on FFT operations, are extremely efficient, but since their intrinsic nature to process blocks of data may suffer of peak misdetection due to the presence of the secondary code in the Galileo BOC(1,1) pilot channel (see Section 3.4.2). In fact, the secondary code acts exactly as the data transition for the GPS signal and it can be the cause of a sign reversal in the correlation operation over the integration interval. For its natural way to look for all the code shifts over all the possible
delays moving along the received signal (Figure 11.1), when the correlation is performed on a single period, the serial acquisition scheme is practically insensitive to the secondary code transitions. In fact the main lobe is identified just when the incoming and the local generated codes are perfectly aligned, hence when the secondary transitions are at the edges of the analyzed stream [42].

![Figure 11.1. Circular correlation for serial search scheme with secondary code](image)

Since the FFT system in frequency domain performs a serial search over the code delay and a parallel search in frequency domain, it results insensitive to the code transition as well as the serial search scheme and it can be used in the acquisition of the L1 Galileo primary code.

The fast acquisition scheme computes an entire row of the search space from a block of data by means of FFT operations. Since it is not possible to know if in the data block the secondary code causes a sign reversal or not, this technique cannot be applied without changes. Figure 11.2 shows how, the secondary code sign reversal within a correlation window makes it no longer periodic; by consequence, the single period circular correlation cannot be used alone to decide the absence or the presence of a signal.

A possible solution to this problem is to conduct a linear correlation as shown in Figure 11.2. Two periods of the incoming signal are correlated with a single period of the local code zero padded to fit the correlation window. The zero terms in the second period does not introduce any advantage in terms of noise reduction generally achieved with a longer integration time. The price to be paid for the fast acquisition scheme to achieve such insensitivity is to perform a linear correlation using two code periods, then to use longer FFTs.

In the following sections, the simulation results of the ROC curves for the different acquisition methods, applied to the Galileo BOC(1,1) modulation, are presented in the case
of integration performed on a single period. The comparison among the schemes, the impact of the number of ADC bits and the comparison with the analytical model developed in Section 10.5 can be appreciated for different values of the false alarm probability and carrier–to–noise ratios.

### 11.4.1 Comparison between the different implementations of the acquisition schemes

In order to compare the three considered implementations of the serial search, fast acquisition and FFT system in frequency domain, the ROC curves for the three schemes have been computed by simulation with the parameters described in Section 11.3; Figure 11.3 shows the comparison for the three techniques.

The three curves are practically equal and if they were shown overlapped on the same figure they would have been indistinguishable, so the three acquisition implementations herein analyzed, even though they perform the correlation or if they proceed in the search space using different strategies, produce the same results. This proves their equivalence as pointed out in Chapter 9.

Figure 11.4 shows the comparison between the conventional fast acquisition scheme and the modified fast acquisition scheme, which performs a linear correlation. The conventional fast acquisition scheme is applied to a situation where the secondary code is not present, while the modified architecture deals with a secondary code transition. The results of Figure 11.4 prove how a linear correlation can be used instead of a circular correlation overcoming the problem of a code transition without any loss in terms of
acquisition performance.

The decision on the presence or absence of a particular code is reliable if it is associated to a small probability of false alarm, then just the left part of the previous graphs are really significant and in order to appreciate better the comparison a zoom of Figure 11.3 is shown in Figure 11.5.

In Tables 11.1 some simulated values for the serial search, the fast acquisition scheme and the FFT in frequency domain acquisition implementations, in the case of integration performed over one code period, are shown and the equivalence among the schemes can be appreciated for different values of the false alarm probability.
11.5 – Comparison between analytical and simulated ROC

In the infinite-bit ADC situation, it is possible to obtain the ROC curves analytically, integrating Equations (10.35) and (10.40), and compare them with the simulation results in order to validate the method for calculating the false alarm and the detection probability. This comparison is depicted in Figure 11.6 in the case of $C_{\text{No}} = 33 \, \text{dB–Hz}$ and integration.
over one code period of the Galileo code. In order to obtain this graph, the simulation for the detection probability has been carried out considering both the Doppler shift and code offset loss uniformly distributed inside the bin width of the search space relative to the perfect alignment situation.

The solid line corresponds to the numerical solution of Equation (10.40) and the marked values to the Montecarlo simulation. The two curves are practically equal so the method used to calculate the false alarm probability and the corresponding detection probability, even if it is not a real statistical analysis, produces the correct results.

The validity of the model can be better appreciated by means of the graph of Figure 11.7, which depicts the detection probability as a function of the carrier–to–noise ratio for a desired false alarm probability, in this case, equal to $P_{fa} = 1 \times 10^{-3}$. Again the solid line corresponds to the model developed in Section 10.5 and the marked values to the Montecarlo simulation. The numerical solution of the detection probability produces the same results obtained by means of simulations proving the validity of the acquisition model studied in this Thesis.

Table 11.2 reports the comparison between the analytical and simulated detection probability as a function of the carrier–to–noise ratio for a probability of false alarm $P_{fa} = 1 \times 10^{-3}$. The equality among the simulation and the model can be appreciated for
11.6 Comparison for different number of ADC bit

When a finite number of ADC bits is used for the signal digitization, it is not generally possible to obtain the analytical expression for the ROC curves. However as reported in

Figure 11.6. Comparison between the analytical ROC and simulated ROC for a CNo = 33 dB–Hz and summation over one code period

Figure 11.7. Comparison between the analytical and simulated detection probability as a function of the carrier–to–noise ratio for a desired false alarm probability $P_{fa} = 1 \times 10^{-3}$

different values of carrier–to–noise ratio.

11.6 Comparison for different number of ADC bit

When a finite number of ADC bits is used for the signal digitization, it is not generally possible to obtain the analytical expression for the ROC curves. However as reported in
Table 11.2. Infinite–bit ADC system: comparison between the analytical ROC and simulated ROC

<table>
<thead>
<tr>
<th>CNo</th>
<th>Acquisition Detection Model $P_d$</th>
<th>Montecarlo Simulated $P_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>$2.82 \times 10^{-3}$</td>
<td>$3.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>25.5</td>
<td>$4.88 \times 10^{-3}$</td>
<td>$5.44 \times 10^{-3}$</td>
</tr>
<tr>
<td>28</td>
<td>$1.016 \times 10^{-2}$</td>
<td>$1.08 \times 10^{-2}$</td>
</tr>
<tr>
<td>30.5</td>
<td>$2.532 \times 10^{-2}$</td>
<td>$2.05 \times 10^{-2}$</td>
</tr>
<tr>
<td>33</td>
<td>$7.192 \times 10^{-1}$</td>
<td>$6.77 \times 10^{-1}$</td>
</tr>
<tr>
<td>35.5</td>
<td>$2.051 \times 10^{-1}$</td>
<td>$2.03 \times 10^{-1}$</td>
</tr>
<tr>
<td>38</td>
<td>$4.811 \times 10^{-1}$</td>
<td>$4.76 \times 10^{-1}$</td>
</tr>
<tr>
<td>40.5</td>
<td>$7.961 \times 10^{-1}$</td>
<td>$7.79 \times 10^{-1}$</td>
</tr>
<tr>
<td>43</td>
<td>$9.623 \times 10^{-1}$</td>
<td>$9.54 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

Section 6.2 the impact of the ADC can be seen as a reduction of the input signal to noise ratio. Figure 11.8 depicts the receiver operative characteristic for the usual carrier–to–noise ratio, CNo = 33 dB–Hz, for 1,2 and 3 ADC bits and a completely floating–point signal representation. The results are obtained by means of Montecarlo simulations and the quantization levels determined to be optimal with respect to the signal dynamic in order to better represent the real front–end AGC behavior.

Figure 11.9 shows the impact of the signal representation, when 1,2 and 3 bit are
Comparison for different number of ADC bit

used for the signal digitization, by means of the graph which depicts the probability of
detection as a function of the CNo for a fixed probability of false alarm \( P_{fa} = 1 \times 10^{-3} \). The results are then compared with the complete floating-point situation.

\[ P_{fa} = 1 \times 10^{-3} \]

The loss of the 1-bit ADC implementation with respect to the non-quantized input signal system is due to the loss of information of the squared signal with respect to a signal with a greater number of bits. This loss is not negligible, as it can be seen from Figure 11.8 and 11.9, but it can be accepted since the 1-bit ADC is much less complex: all the multiplications can be performed with a two-by-two matrix and no AGC is required. It has to be remarked that the floating-point input signal is an extreme case of ADC in which no quantization takes place, so that a real system using an ADC with a number of bits greater than one shows intermediate performances.

The price to be paid for this reduction in complexity is the loss in the detection probability. Figures 11.8 and 11.9 show, according to the analysis pointed out in Section 6.2, how 3-bits resolution provides close to minimum SNR degradation and signal distortion, which leads to a relative detection probability comparable to the floating-point condition.

In Table 11.3 some simulated values are shown in the case of the integration performed over one code period, where the loss analyzed in this section is highlighted for different values of the CNo.

From the results addressed in this section, in the following a complete floating-point system will be considered to represent a front-end with an ADC with 3-bits resolution or higher.
11 – Acquisition of the Galileo BOC(1,1) modulation

<table>
<thead>
<tr>
<th>CNo</th>
<th>1-bit ADC</th>
<th>2-bits ADC</th>
<th>3-bits ADC</th>
<th>Floating-point</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>$2.88 \times 10^{-3}$</td>
<td>$2.24 \times 10^{-3}$</td>
<td>$2.34 \times 10^{-3}$</td>
<td>$3.20 \times 10^{-3}$</td>
</tr>
<tr>
<td>25.5</td>
<td>$2.99 \times 10^{-3}$</td>
<td>$4.80 \times 10^{-3}$</td>
<td>$4.69 \times 10^{-3}$</td>
<td>$5.44 \times 10^{-3}$</td>
</tr>
<tr>
<td>28</td>
<td>$6.40 \times 10^{-3}$</td>
<td>$7.53 \times 10^{-3}$</td>
<td>$1.01 \times 10^{-3}$</td>
<td>$1.08 \times 10^{-3}$</td>
</tr>
<tr>
<td>30.5</td>
<td>$1.16 \times 10^{-2}$</td>
<td>$1.79 \times 10^{-2}$</td>
<td>$2.28 \times 10^{-2}$</td>
<td>$2.05 \times 10^{-2}$</td>
</tr>
<tr>
<td>33</td>
<td>$3.55 \times 10^{-2}$</td>
<td>$5.44 \times 10^{-2}$</td>
<td>$6.48 \times 10^{-2}$</td>
<td>$6.77 \times 10^{-2}$</td>
</tr>
<tr>
<td>35.5</td>
<td>$9.50 \times 10^{-2}$</td>
<td>$1.69 \times 10^{-1}$</td>
<td>$1.99 \times 10^{-1}$</td>
<td>$2.03 \times 10^{-1}$</td>
</tr>
<tr>
<td>38</td>
<td>$2.53 \times 10^{-1}$</td>
<td>$4.03 \times 10^{-1}$</td>
<td>$4.72 \times 10^{-1}$</td>
<td>$4.76 \times 10^{-1}$</td>
</tr>
<tr>
<td>40.5</td>
<td>$5.43 \times 10^{-1}$</td>
<td>$7.22 \times 10^{-1}$</td>
<td>$7.75 \times 10^{-1}$</td>
<td>$7.79 \times 10^{-1}$</td>
</tr>
<tr>
<td>43</td>
<td>$8.37 \times 10^{-1}$</td>
<td>$9.32 \times 10^{-1}$</td>
<td>$9.51 \times 10^{-1}$</td>
<td>$9.54 \times 10^{-1}$</td>
</tr>
</tbody>
</table>

Table 11.3. Number of ADC bits: comparison between the simulated ROC

11.7 Simulation results for multiple period integration

In order to increase the detection probability for a given false alarm probability, a summation over more than one code period can be performed. In this case, the threshold value has to be increased according to Equations (10.35), in which the number of samples used for the integration, $N$, and the number of non-coherent summations, $K$, appear.

The length of a data record used for the summation in an acquisition scheme is limited by two factors, the navigation data or secondary code transitions and the Doppler effect on the spreading code.

The presence of a navigation data or secondary code transitions in the data record causes a spreading effect of the output spectrum and the performances of the acquisition system are degraded. For the GPS C/A signal on the L1 carrier, the navigation data rate is 50 bit/s (see reference [4]), so that the length of a data bit is 20 ms, i.e. 20 periods of the spreading code. The maximum data record that can be used for the coherent summation is, therefore, 10 ms or 10 C/A code periods. In fact, in a 20 ms time interval only one navigation data transition can occur. Then, if there is a transition in the first 10 ms data record, the second 10 ms will be transition free. On the other hand, the sequence length cannot be less than a code period or 1 ms and even in this minimum interval a data transition can occur. In order to guarantee no data transition, the acquisition algorithm should take into account two consecutive data records of equal duration, but less than 10 ms, perform the coherent summation over these two data records and then declare the detection if one of the two envelopes or both of them exceed the threshold. Unfortunately the presence of the secondary code on the Galileo signal does not allow the possibility to perform the acquisition of consecutive pieces of signal, since every period of the primary code is modulated by the secondary short code, then it is not guaranteed the absence
of a secondary code transition in the following integration period. By the way, in order to increase the detection probability, a summation over than one code period in a non-coherent way can be applied, accepting the squaring loss due to the square operation performed prior the envelope detector.

The other factor that limits the data record length is the Doppler effect on the spreading code. The tracking system that follows the acquisition system cannot work when the local replica of the spreading code is misaligned of more than half a chip with respect to the incoming code. Since the maximum expected Doppler shift on the Galileo code is $2.64 \text{ chip/s}$, as shown by Equation (4.13), two frequencies different by $2.64 \text{ Hz}$ take about $189 \text{ ms}$ to change by half a chip, i.e. about $47$ code periods.

The limit imposed by the Doppler effect on the spreading code is by far higher than the one imposed by the presence of navigation data or the secondary code on the Galileo signal, so the acquisition methodology which will be addressed in this section is a non-coherent summation on the primary code period, and once the begging of the primary code has been estimated the starting point of the secondary code can be easily recovered. This is a hierarchal acquisition strategy, which is similar to the one used in the UMTS communication scenarios.

Figure 11.10 shows the ROC curve for a $\text{CNo} = 33 \text{ dB–Hz}$ at the increase of the number of summed code periods $K$ for the infinite–bit ADC. The lines represent the results obtained from the model described by the Equation (10.40), while the marked values are the Montecarlo simulation results.

The increased performance in terms of detection probability due to the increased...
number of integration period can be better quantified by means of Figure 11.11, where the detection probability as a function of the input CNo is depicted.

![Detection probability versus input CNo for P_{fa} = 1 \times 10^{-3} for K from 1 to 5](image)

Figure 11.11. Comparison between the analytical and simulated detection probability as a function of the carrier–to–noise ratio for a desired false alarm probability \( P_{fa} = 1 \times 10^{-3} \) from one up to five code periods

The results agree with the theory carried out in Chapter 10. According to the theory, the gain in terms of signal–to–noise ratio at the envelope detector input when more than a code period is used in the integration process leads to better acquisition performance in terms of signal detection capability and the price to be paid is a longer acquisition time.

In order to appreciate the detection probability gain, in Table 11.4 some values of \( P_d \) for an integration time from one up to five code periods are shown. Comparing the single results, it can be seen how it is possible to achieve the required performance at a certain CNo increasing the number of integration periods before the acquisition decision.

All the Montecarlo simulations have been performed over an adequate number of samples, so as to count at least 100 "events" for each simulated point: \( 5 \times 10^5 \) samples, according to the expected missed detection probability, have been used for the detection probability.

Finally, Figure 11.12 shows the detection probability for integration time from one to fifty non–coherent integrations for the usual probability of false alarm \( P_{fa} = 1 \times 10^{-3} \). It is easy to see, how the gain tends to be smaller as \( K \) becomes larger.
11.8 Galileo BOC(1,1) side lobes effect on the acquisition performance

The forecoming BOC modulation foreseen for the Galileo Signal In Space (SIS) presents a high degree of spectral separation from conventional GPS signals and it also enhances multipath rejection and improves the accuracy on the position estimation. Although

![Image](image_url)
more precise, BOC modulation increases the receiver complexity and brings some draw-
backs associated with the characteristic shape of its autocorrelation function. The pres-
ence of the side lobes in the autocorrelation function (see Section 3.6), the longer code
length and the higher rate lead to a more complex acquisition process.

Multiple positive and negative peaks make a receiver more sensitive to the dynamic
conditions and they introduce the risk of miss-detection of the main peak and the wrong
peak selection. The receiver must ensure a non ambiguous peak selection and a non
ambiguous signal tracking. This becomes more complex in the presence of noise and
multipath where acquiring and maintaining the tracking locked on the correct peak are
extremely difficult tasks.

Due to the presence of the side lobes there is a non negligible probability that the
receiver will become locked onto the wrong peak, which corresponds to an incorrect
time-of-arrival (TOA) estimate. A significant bias of approximately 150 m would then
present on the range measurements. The presence of a secondary peaks can cause a
receiver to lock on a side peak, which is unacceptable for navigation applications. This
problem can be handled both at the acquisition and tracking level. In this section the
problem of the side peaks acquisition will be investigated and a strategy ables to reduce
the declaration of the signal presence in correspondence of a side lobe proposed.

Figure 11.13 shows the nominal value of the autocorrelation function of the BOC(1,1)
code calculated over one code period when the sampling frequency is about twice the
slot rate. Seven samples are considered: the main peak \(C_0\), which corresponds to the
perfect alignment of the local code with the received one, and four samples on both sides
of the main peak which are named as depicted in graph 11.13. It is here highlighted
that the position of the left and right side peaks correspond to the samples \(C_{-2}\) and \(C_2\)
respectively (see Figure 11.13).

In order to formalize this analysis the following definition will be assumed:

- **Main lobe detection probability** as the probability that the current investigated cell
corresponds to the position of the maximum value of the autocorrelation function
and the signal is declared present

- **Side lobe detection probability** as the probability that the current investigated cell cor-
responds to the position of a side lobe of the autocorrelation function and the signal
is declared present

In order to take into account the particular shape of the Galileo BOC(1,1) correlation
matrix a three cells strategy has been analyzed. According to Figure 11.13 naming \(\hat{C}_0\) the
measured sample at the current code delay bin, the detection is declared when the sample
\(\hat{C}_0\) exceeds the threshold \(V_t\) and if it is respectively greater than the side samples \(\hat{C}_2\) and
\(\hat{C}_{-2}\), which represents the measured values corresponding to the possible position of the
side lobes of the BOC(1,1) correlation function.
In other words the decision rule becomes
\[
\left( \tilde{C}_0 \geq V_t \text{ AND } \tilde{C}_0 \geq \tilde{C}_{-2} \text{ AND } \tilde{C}_0 \geq \tilde{C}_2 \right)
\]  
(11.5)

The strategy consists, in practice, of a moving windows (see Figure 11.14), which move along the search space in the direction of the code delay shift. When the decision rule of Equation (11.5) is satisfied the tracking loop is initialized with the code delay of the central cell $\tilde{C}_0$.

Denoting with the symbol $\//-$ the event “the cell is aligned to” the side lobe detection probability $p_{ds}$ corresponds to the following situation
\[
p_{ds} = P\left[\tilde{C}_0 > V_t \mid \tilde{C}_0 /\tilde{C}_{-2} \text{ or } \tilde{C}_0 /\tilde{C}_{-2}, \max(\tilde{C}_0, \tilde{C}_{-2}, \tilde{C}_2) = \tilde{C}_0 \right]
\]
and analogously, the main lobe detection probability $p_{dm}$ to

$$p_{dm} = P \left[ \hat{C}_0 > V_t \mid \hat{C}_0/|C_0|, \max(\hat{C}_0, \hat{C}_{-2}, \hat{C}_2) = \hat{C}_0 \right]$$

Figure 11.15 shows the ideal situation, in absence of noise, when the current sample $\hat{C}_0$ falls on a side lobe and when it corresponds to the main correlation lobe. Since the side lobes are not local maximums of the triple $\hat{C}_{-2}, \hat{C}_0$ and $\hat{C}_2$, the result expected is a higher rejection capability of locking the tracking loop on a side lobe, which may correspond to a wrong acquisition code delay.

This technique will be addressed, in the following, as local max single dwell acquisition strategy, since the decision is taken just if the current cell bin overcome the threshold and at the same time it results to be a local maximum.

The simulations have been performed using $10^6$ samples, in order to obtain reliable results. In Figure 11.16 the comparison between the receiver operative characteristic obtained with the classical acquisition strategy and the modified local maximum strategy is carried out for a CNo = 33 dB–Hz and for one and five integration periods respectively. The dash–line refers to the side lobes detection probability for the local maximum strategy while the dash–dotted line for the conventional architecture. In the same figure the main lobe detection probability for the different strategy is depicted: the solid line refers to the classical architecture while the marked values to the three cells strategy.

Figure 11.16 and 11.17 show the main and side lobe detection probabilities comparison for the classical and the local maximum strategies. The two implementations are almost equivalent concerning the main lobe detection probability. Better performance can be achieved with the classical technique, since due to the noise presence the further
test to verify if the current cell is also a local maximum may reduce the declaration of the signal presence. Different is the situation of the side lobe detection, in fact as expected, the local maximum strategy is able to reduce the detection of the signal corresponding to a side lobe code delay position. Moreover it has to be highlighted how increasing the integration time the side lobe detection probability increases according to the higher value of the signal-to-noise ratio for the conventional architecture, while the trend is the opposite for the local maximum strategy.

Figure 11.17. Main lobe detection probability comparison for the two different architectures; solid lines for the classical architecture and marked values for the local maximum strategy.
In Figure 11.18 the same analysis, as a comparison on the basis of the detection probability of the side lobes as a function of the CNo for a fixed false alarm probability, is carried out. The classical and the local maximum strategies are compared for one and five integration time. As for the case of Figure 11.16, the solid line refers to the side lobes detection probability obtained with the local maximum strategy and the dashed line to the side lobe detection probability for the classical architecture.

While the side lobe detection probability for the classical acquisition scheme has the same trend of the main lobe detection probability, Figure 11.18 shows how for the local maximum strategy is driven to zero increasing both the integration time and the CNo without changing the system performance in terms of detection of the main lobe, as shown in Figure 11.17. In order to better underline the results drawn in Figure 11.18, the side lobe detection probability \( P_{ds} \) for the classical and local maximum strategies are reported in Table 11.5 for a single period integration time and in Table 11.6 for five respectively.

It is also interesting to see the particular "bell–shape" of the side lobe detection probability when the local maximum strategy is adopted. In Figure 11.19 the side lobe detection probability for the local maximum strategy is shown for the case of one and five integration periods.

At the increasing of the carrier–to–noise ratio the detection of a side lobe increases. This is mainly due to the detection of a local maximum due to the presence of the noise contribution, which buries the signal level. When the signal–to–noise ratio is sufficiently high to allow the signal to rise the noise floor, the three cells algorithm can successfully detect a local maximum and estimate the right main peak position in the search space.
Table 11.5. Side Lobe Detection probability for one integration time: Local maximum and classical strategy comparison

<table>
<thead>
<tr>
<th>CNo</th>
<th>Classical Strategy</th>
<th>Local Maximum Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_d )</td>
<td>( P_{ds} )</td>
</tr>
<tr>
<td>23</td>
<td>( 3.2 \times 10^{-3} )</td>
<td>( 6.4 \times 10^{-4} )</td>
</tr>
<tr>
<td>25.5</td>
<td>( 5.44 \times 10^{-3} )</td>
<td>( 1.92 \times 10^{-3} )</td>
</tr>
<tr>
<td>28</td>
<td>( 1.08 \times 10^{-3} )</td>
<td>( 2.45 \times 10^{-3} )</td>
</tr>
<tr>
<td>30.5</td>
<td>( 2.05 \times 10^{-2} )</td>
<td>( 4.90 \times 10^{-3} )</td>
</tr>
<tr>
<td>33</td>
<td>( 6.77 \times 10^{-2} )</td>
<td>( 6.40 \times 10^{-3} )</td>
</tr>
<tr>
<td>35.5</td>
<td>( 2.03 \times 10^{-1} )</td>
<td>( 1.55 \times 10^{-2} )</td>
</tr>
<tr>
<td>38</td>
<td>( 4.76 \times 10^{-1} )</td>
<td>( 3.77 \times 10^{-2} )</td>
</tr>
<tr>
<td>40.5</td>
<td>( 7.79 \times 10^{-1} )</td>
<td>( 1.41 \times 10^{-1} )</td>
</tr>
<tr>
<td>43</td>
<td>( 9.54 \times 10^{-1} )</td>
<td>( 2.90 \times 10^{-1} )</td>
</tr>
</tbody>
</table>

Table 11.6. Side Lobe Detection probability for five integration time: Local maximum and classical strategy comparison

<table>
<thead>
<tr>
<th>CNo</th>
<th>Classical Strategy</th>
<th>Local Maximum Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_d )</td>
<td>( P_{ds} )</td>
</tr>
<tr>
<td>23</td>
<td>( 5.86 \times 10^{-3} )</td>
<td>( 1.49 \times 10^{-3} )</td>
</tr>
<tr>
<td>25.5</td>
<td>( 1.53 \times 10^{-2} )</td>
<td>( 2.66 \times 10^{-3} )</td>
</tr>
<tr>
<td>28</td>
<td>( 5.57 \times 10^{-2} )</td>
<td>( 4.48 \times 10^{-3} )</td>
</tr>
<tr>
<td>30.5</td>
<td>( 1.71 \times 10^{-1} )</td>
<td>( 8.53 \times 10^{-3} )</td>
</tr>
<tr>
<td>33</td>
<td>( 4.71 \times 10^{-1} )</td>
<td>( 2.47 \times 10^{-2} )</td>
</tr>
<tr>
<td>35.5</td>
<td>( 8.00 \times 10^{-1} )</td>
<td>( 9.14 \times 10^{-2} )</td>
</tr>
<tr>
<td>38</td>
<td>( 9.63 \times 10^{-1} )</td>
<td>( 2.80 \times 10^{-1} )</td>
</tr>
<tr>
<td>40.5</td>
<td>( 9.98 \times 10^{-1} )</td>
<td>( 5.41 \times 10^{-1} )</td>
</tr>
<tr>
<td>43</td>
<td>( 9.99 \times 10^{-1} )</td>
<td>( 7.33 \times 10^{-1} )</td>
</tr>
</tbody>
</table>

This change in the detection probability leads to the particular bell–shape of the curves of Figure 11.17:

### 11.9 Conclusion

In this chapter the conventional algorithms already used for the acquisition of the GPS signal have been investigated under the light of a possible employment for the acquisition of the Galileo BOC(1,1) modulation. From the previous analysis it is possible to conclude that the differences in the signal definitions of the Galileo BOC(1,1) have to be
taken into account in order to declare the presence of a particular code of interest in a reliable way. The main differences are the presence of a secondary code, which acts in the same way as the GPS navigation data, and the particular shape of the correlation function and the presence of side lobes.

The secondary code may introduce a sign reversal in the data record with a reduction of the efficiency of the circular correlation performed by the fast acquisition technique, the problem can be overcome employing a linear correlation with a consequent increment of system complexity due to the longer number of samples that have to be processed. Moreover, the sign reversal introduced by the secondary code does not allow to increase the integration time in a coherent way in order to increase the detection probability performance. A non–coherent strategy has to be used and the price to be paid is the squaring loss of the summation prior to the envelope detector.

The presence of two side lobes in the correlation shape of the BOC(1,1) signal may lead to a non negligible probability that the receiver will become locked onto the wrong peak, which corresponds to an incorrect time-of-arrival (TOA) estimate. In this chapter a possible strategy, which takes into account the particular shape of the BOC(1,1) correlation function, has been proposed and investigated. The strategy called “local maximum technique” based on a three cells decision has proved to be able to reduce the side lobe detection probability without reducing the main lobe detection capability. This is obtained without a significant increase of the system complexity as in the case of the “bump–jumping” technique proposed in reference [43] or the 3 dB system loss of the “BPSK-Like” technique of reference [44].
Part IV

Signal Tracking
Chapter 12

Signal Tracking

After the acquisition phase has brought the received and the locally generated code within less than a half chip period residual offset, a fine synchronization, named tracking, takes over and continuously maintains and corrects the best possible alignment between the two codes by means of closed loop operations. This Chapter and the following one deal with the methods for tracking the satellite navigation system signal; the whole tracking process is a two-dimensional (code and carrier) signal replication process. In this Chapter more emphasis goes to the code tracking, while just a brief introduction to the carrier tracking will be given. In order to be able to better understand the behavior of the modified tracking architecture presented in Chapter 16, with Quality Control features, the linear model of a completely digital Delay Locked Loop will be addressed in this chapter.

12.1 Early–Late code tracking loops

Figure 12.1 shows a high-level block diagram of a typical satellite navigation system receiver. It consists of a Delay Lock Loop for code tracking and a Costas Loop for carrier tracking. Obviously an \( n \) parallel channel receiver will have \( n \) sets of blocks corresponding to each independent tracking loop. In a receiver, the digitized IF signal is input to each of these parallel channels. The input signal is beat with the locally generated in-phase and quadrature-phase replicas of the carrier. The signal is then correlated with the prompt (P), early (E) and late (L) versions of the locally generated code, and the correlation values are integrated for a pre-detection integration period [4]. The distance between Early, Prompt and Late codes is generally called correlator spacing. The early and late correlation values in the in-phase and quadrature-phase arms (IE, IL, QE, QL) are generally used for code tracking, whereas the prompt correlation values (IP, QP) are used for carrier tracking. Some code discriminators, such as the dot-product, use prompt
correlation values as well, and they can be also used as lock detector. The correlation values are used by the discriminator functions in the loop filters. The code and carrier loop filters generate corrections to the locally generated code and carrier respectively, to maintain the discriminator function output around zero. The replica carrier (including carrier Doppler) signals are synthesized by the carrier numerical controlled oscillator (NCO) and the discrete sine and cosine mapping function. The replica codes are synthesized in a similar way by the code generator, a 3-bit shift generator and the code NCO [4]. In the following sections features of the code tracking loop are analyzed in details, discussing their adaptation to deal with the Galileo signals.

Figure 12.1. Generic digital receiver channel block diagram

12.1.1 Delay Lock Loop (DLL)

Figure 12.2 illustrates the block diagram of the receiver code tracking loop. This last scheme is characterized by the design of the programmable predetection integrators, the code loop discriminator and the code loop filter; indeed these three functions play a key role in determining the performance characteristics of the receiver.

The digital DLL needs to be designed in order to track, in presence of white Gaussian noise, the input PRN code phase dynamics due to the relative motion between the satellite and the receiver. Performance are usually evaluated in terms of root-mean-square (rms) tracking jitter.
12.1 – Early–Late code tracking loops

12.1.2 Predetection Integration

The predetection stage is a processing block operating after the signal has been down-converted to baseband by the carrier and code stripping process, but prior to be passed through a signal discriminator. The system requires three complex correlators in order to produce three in-phase components and three I-Q components all integrated and dumped to produce IE, IP, IL, and QE, QP, QL [45].

The integrate and dump accumulators provide filtering and re-sampling at the processor baseband input rate; such a rate can be higher or lower depending on the desired predetection bandwidth, but the predetection integration must satisfy the two following constraints:

- it must be lower than an entire navigation data bit duration if no data wipe off process is implemented (unless a pilot channel is used, with no data);

- it must be greater than the PRN code period duration to exploit the PRN code correlation properties.
12.1.3 Code Loop Discriminator

The code phase error of the incoming signal can be tracked using a coherent discriminator or a non-coherent discriminator. To operate a coherent loop, the code discriminator depends on the quality of the carrier phase tracking in order to produce precise code phase errors. Since the carrier tracking loop is one of the most critical blocks in terms of the receiver’s dynamic stress threshold, the use of a non-coherent loop is obviously a benefit for tracking signals because of its more robust behavior in low-cost mass-market receivers [4, 45].

The following list reports some of the most common non-coherent DLL discriminators [4]

- **Dot product power**
  \[
  D_P = \sum (I_E - I_L)I_P + \sum (Q_E - Q_L)Q_P
  \]
  This discriminator uses all three correlators and this results in the lowest baseband computational load. For $\frac{1}{3}$ chip correlator spacing, it produces nearly true error output within $\pm \frac{1}{3}$ chip of input error.

- **Early minus late power**
  \[
  D_{EMLP} = \sum \left[ (I_E)^2 + (Q_E)^2 \right] - \sum \left[ (I_L)^2 + (Q_L)^2 \right]
  \]
  Moderate computational load. Essentially the same DLL discriminator error performance as early minus late envelope $\pm \frac{1}{3}$ chip of input error.

- **Early minus late envelope**
  \[
  D_{EMLE} = \sum \sqrt{(I_E)^2 + (Q_E)^2} - \sum \sqrt{(I_L)^2 + (Q_L)^2}
  \]
  Higher computational load. For $\frac{1}{3}$ chip correlator spacing, produces good tracking error within $\pm \frac{1}{3}$ chip chip of input error.

- **Early minus late envelope normalized**
  \[
  D_{EMLN} = \frac{\sum \sqrt{(I_E)^2 + (Q_E)^2} - \sum \sqrt{(I_L)^2 + (Q_L)^2}}{\sum \sqrt{(I_E)^2 + (Q_E)^2} + \sum \sqrt{(I_L)^2 + (Q_L)^2}}
  \]
  The normalization by the early plus late envelope is performed to remove amplitude sensitivity. Highest computational load. For $\frac{1}{3}$ chip correlator spacing, produces good tracking error within less than $\pm \frac{1}{3}$ chip of input error.
The presence of the two side lobes in the Galileo BOC(1,1) signal affects the shape of the discriminator function and possible chip correlator spacing. In fact, the squaring operation used in the EMLN discriminator makes the two side lobes positive reducing the width of the main peak of the correlation function to $\frac{1}{3}$ of chip. As a result, the EMLN requires a correlator spacing lower than $\frac{1}{3}$ chip in order to function properly.

Figure 12.3 compares the so called $S$–curves for the four considered discriminators for a front–end infinite bandwidth.

The curves shown in the figure have been obtained in absence of input noise; the presence of noise and the limitation due to a finite bandwidth tend to flatten the slopes and round the edge of the discriminators.

12.2 Carrier Tracking Loop

Figure 12.4 illustrates the block diagram of the GPS/Galileo receiver carrier tracking loop. The receiver carrier tracking loop is characterized by the designs of the carrier predetection integrators, of the carrier loop discriminators and of the carrier loop filters. These three functional blocks determine the performance characteristics of the receiver in terms of the carrier loop thermal noise error and of the maximum line–of–sight dynamic stress threshold. Since the carrier tracking loop is always one of the most sensitive blocks in a stand-alone receiver, its threshold characterizes the GPS/Galileo receiver performance [4].

The carrier loop processing is performed correlating the input signal with the local
code replicas. The correlators are followed by a post-correlation processing block. Such a block accumulates the correlated signal components over a Predetection Integration Time (PIT) $T_S$.

The unaided carrier loop discriminator defines the type of tracking loop as a PLL, a Costas PLL (which is a PLL type discriminator that tolerates the presence of data modulation on the baseband signal), or a Frequency Lock Loop (FLL). Different discriminators have different features: the PLL and the Costas Loop are the most accurate but are more sensitive to dynamic stress than FLL. The PLL and Costas Loop discriminators produce phase errors at their outputs, while the FLL discriminator produces a frequency error $\Delta f$.

In order to deal with SIS carrying data (i.e., not pilot channels) only Costas carrier tracking loops can be used because of the navigation message data modulation. Costas loops are insensitive to $180^\circ$ degree phase reversal in the I and Q samples if the PIT of the I and Q signals do not straddle the data bit transitions.

References [14, 45] summarize several Costas PLL discriminators, which can be used in a GPS/Galileo receiver. The first two discriminators are identical to those used in a baseline PLL. The two–quadrants ATAN function PLL discriminator remains linear over the full input error range of $\pm 90^\circ$ deg. The main discriminators used in a PLL are listed below.

- $\text{sign}(I_{PS} \cdot Q_{PS})$
  
  Near optimal at high SNR characterized by a Phase error equal to $\sin \phi$. Slope proportional to signal amplitude $A_{IN}$. Least Computational burden.

- $I_{PS} \cdot Q_{PS}$
  
  Near optimal at low SNR with phase error approximately equal to $\sin 2\phi$. Slope
12.3 Principles of digital feedback systems

Figure 12.5 shows the well known digital model of a DLL/PLL feedback system as reported in reference [46]. It is the linearized digital closed-loop carrier or code synchronizer which allows to erase the error signal $e[n]$. The expression of $e[n]$ depends on the carrier phase estimator that is used.

A digital PLL or equivalently a digital DLL is based on the following principle: assuming the incoming signal phase is $\xi[n]$, if the estimated phase is $\hat{\xi}[n] > \xi[n]$, then a negative error $e[n]$ is generated by the Phase Discriminator, so that $\hat{\xi}[n+1]$ is reduced at the Numeric Controlled Oscillator (NCO) output. On the other hand, if $\hat{\xi}[n] < \xi[n]$, a positive value of $e[n]$ is generated, increasing $\hat{\xi}[n+1]$ at the output of NCO. The error signal depends on the phase error $\phi[n] = \hat{\xi}[n] - \xi[n]$ and the S-curve $S(\phi)$ is defined as the average value of the error signal under open-loop conditions. A digital phase locked loop correctly works if the S-curve is an odd function with a positive slope in the origin. The analysis of the DLL is usually performed over a linearized model obtained considering the loop working in the linear region of the S-curve close to the origin and then studying the behavior of the loop for small errors. If the approximation is valid, then:

$$ e[n] \simeq \frac{dS(\phi)}{d\phi} \bigg|_{\phi=0} \phi[n] + n_w[n] \quad (12.1) $$
where $n_w[n]$ represents the thermal noise and the data modulation. Equation (12.1) shows that the output of the phase discriminator consists of both the error $\phi[n]$ due to the incoming signal phase dynamics and the error $n_\phi[n]$ due to the incoming additive noise.

### 12.3.1 Digital DLL Tracking performance in the absence of noise

In the case of small code phase tracking error, the phase discriminator can be modeled as Equation (12.1) (see [46]), where $\frac{dS(\phi)}{d\phi}|_{\phi=0}$ is the slope of the discriminator at the origin.

Let $F(z)$ be the loop filter transfer function, $D(z) = \frac{z^{-1}}{1 - z^{-1}}$ is a transfer function which represents the functionality of the NCO in the digital model of a generic DLL. The normalized phase delay estimate can be obtained based on the output of the code phase discriminator as

$$\hat{\xi}(z) = N(z)F(z)e(z)$$  \hspace{1cm} (12.2)

The loop transfer function for the input code phase and the equivalent noise can be easily obtained from the scheme of Figure 12.5, with the approximation $D(z) = K$ according to Equation (12.1) and substituting $\phi(z) = \xi(z) - \hat{\xi}(z)$ into Equation (12.2). After some algebraic manipulation it is possible to write

$$H_\xi(z) = \frac{z-1}{KF(z) + (z-1)}$$

$$H_N(z) = \frac{KF(z)}{KF(z) + (z-1)}$$  \hspace{1cm} (12.3)

and the tracking error $\phi(z)$ as

$$\phi(z) = H_\xi(z)\xi(z) - H_N(z)\frac{N_w(z)}{K}$$  \hspace{1cm} (12.4)

It is obvious from Equation (12.4) that the tracking error $\phi[n]$ consists of two components, one due to the system dynamics represented by the first term and the other due to the input noise represented by the second term.

The steady tracking error loop is related to the loop order, which depends on the loop filter $F(z)$ transfer function.

- **1st order loop**

  A stable 1st–order loop, is a tracking loop characterized by a loop filter transfer function $F(z) = g_1$ and a loop gain $G_1 = Kg_1$. It can track a phase step input with steady tracking error $\phi[\infty] = 0$ and a frequency step input $\xi[n] = a \cdot U[n]$ with $\phi[\infty] = \frac{a}{G_1}$. 


• 2nd order loop

A second order loop is identified by a loop filter transfer function $F(z) = g_1 + \frac{g_2}{1-z^{-1}}$ and loop gains $G_1 = K g_1$ and $G_2 = K g_2$. It can track a phase step input and a frequency step input with steady tracking error $\phi[\infty] = 0$.

A necessary and sufficient condition of system stability is that the poles of the system function must lie inside the unit circle of the $z$–plane \([47]\), therefore, the stable range of loop gains can be easily proved to be $0 < G_1 < 2$ for a 1st order loop, $r > 1$ and $0 < G_1 < \frac{4}{r-1}$ for a 2nd order loop where $r = 1 + \frac{G_1}{G_2}$.

12.3.2 Digital DLL tracking performance in presence of noise

In order to derive the performance of the tracking loop the noise contribution $n_w[n]$ must be evaluated. Figure 12.6 shows the functional model of the tracking loop discriminator output which will be used in this analysis. For the sake of simplicity just the noise contribution of the Early–minus–Late discriminator output will be derived.

![Figure 12.6. Delay lock loop to track code phase](image)

The resulting difference of the Early and Late codes producing the discriminator output, according to Figure 12.6, can be written as

$$S = S_E - S_L + N_E - N_L$$

where $N_E - N_L$ represents the noise contribution at the discriminator output. The noise mean value can be calculated evaluating $E\{N_E - N_L\} = E\{N_E\} - E\{N_L\}$, then

$$E\{N_E\} = \frac{1}{A_{IN}} \sum_{n=0}^{N-1} E\{n_w[n]\} x_{LOC}[n + \hat{\xi} - \tau_E] \quad (12.5)$$

being $n_w[n]$ the noise sample and $\tau_E$ the Early code shift. From Equation (12.5) it is easy to state that the mean value of the noise contribution at the discriminator output $N_E$ is
equal to zero being the noise sample \( n_w[n] \) a zero mean gaussian noise variable, and in the same way, it can be proved that \( N_L = 0 \).

The noise signal power at the output of the discriminator is given by

\[
\sigma_{N_D}^2 = E \{ (N_E - N_L)^2 \} = E \{ N_E^2 - 2N_E N_L + N_L^2 \} \tag{12.6}
\]

which consists of three different components: \( E \{ N_E^2 \}, \ E \{ N_L^2 \} \) and \( E \{ N_E N_L \} \).

\[
E \{ N_E^2 \} = \frac{1}{A_{IN}^2} \frac{1}{A_{IN} N^2} \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} E \left\{ n_w[n] n_w[l] x_{LOC}[n + \hat{\xi} - \tau_E] x_{LOC}[l + \hat{\xi} - \tau_E] \right\} = \frac{1}{A_{IN}^2} \frac{1}{A_{IN} N^2} \sum_{n=0}^{N-1} E \left\{ n_w^2[n] \right\} x_{LOC}^2[n + \hat{\xi} - \tau_E] = \frac{1}{A_{IN}^2} \frac{1}{A_{IN} N^2} \sigma_n^2 \tag{12.7}
\]

where Equation (12.7) has been obtained using the property of the independent gaussian noise variables

\[
E \left\{ n_w[n] n_w[l] \right\} = \begin{cases} \sigma_n^2 & \text{if } n = l; \\ 0 & \text{if } n \neq l; \end{cases}
\]

and in the same way \( E \{ N_L^2 \} \) can be obtained

\[
E \{ N_L^2 \} = \frac{1}{A_{IN}^2} \frac{1}{A_{IN} N^2} \sigma_n^2 \tag{12.8}
\]

However, it has still not been found the influence of the mixed term \( E \{ N_E N_L \} \),

\[
E \{ N_E N_L \} = \frac{1}{A_{IN}^2} \frac{1}{A_{IN} N^2} \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} E \left\{ n_w[n] n_w[l] x_{LOC}[n + \hat{\xi} - \tau_E] x_{LOC}[l + \hat{\xi} - \tau_L] \right\} = \frac{1}{A_{IN}^2} \frac{1}{A_{IN} N^2} \sum_{n=0}^{N-1} E \left\{ n_w^2[n] \right\} x_{LOC}[n + \hat{\xi} - \tau_E] x_{LOC}[n + \hat{\xi} - \tau_L] = \frac{1}{A_{IN}^2} \frac{1}{A_{IN} N^2} \sigma_n^2 \sum_{n=0}^{N-1} x_{LOC}[n + \hat{\xi} - \tau_E] x_{LOC}[n + \hat{\xi} - \tau_L] = \frac{1}{A_{IN}^2} \frac{1}{A_{IN} N^2} \sigma_n^2 R(\tau_E - \tau_L) \tag{12.9}
\]

and finally it is possible to obtain the noise power contribution at the discriminator output putting all the pieces together

\[
\sigma_{N_D}^2 = \frac{2\sigma_n^2}{A_{IN}^2} [1 - R(\tau_E - \tau_L)] \tag{12.10}
\]
12.3 – Principles of digital feedback systems

12.3.3 Equivalent delay lock loop noise bandwidth

The total output noise of the discriminator can be still considered white noise due to the "dump" operations in the "Average and Dump" low–pass filter. Under this hypothesis, if the noise samples \( n_w[n] \) are stationary with mean zero the steady state variance of \( \phi[n] \) is given by

\[
\sigma^2_\phi = \frac{1}{2\pi j} \oint_{|z|=1} H_N(z) H_N(z^{-1}) z^{-1} R_N(z) dz \tag{12.11}
\]

where

\[ R_N(z) = \mathcal{Z}\{E(n_w[n]n_w[n+k])\} \]

is the z–transform of the noise autocorrelation normalized to the square value of the discriminator slope \( K^2 \). Equations (12.10) and (12.5) state that it is possible to model \( n_w[n] \) to be uncorrelated zero mean with variance \( \sigma^2_{N_D} \), so that Equation (12.11) can be written as

\[
\sigma^2_\phi \approx \frac{2\sigma^2_n}{A_{IN} N K^2} [1 - R(\tau_E - \tau_L)] \frac{1}{2\pi j} \oint_{|z|=1} H_N(z) H_N(z^{-1}) z^{-1} dz \tag{12.12}
\]

The model in Figure 12.2 assumes that the received signal has been processed by the front–end and down-converted to baseband by the PLL, thus the received signal does not include an RF carrier. The received signal power is then \( P = A_{IN}^2 \) and the noise power, according to the notation introduced in Chapter 4, is \( \sigma^2_n = N_0 B_s \).

Defining the input power spectral density as

\[ N_W = \frac{N_0}{K^2} [1 - R(\tau_E - \tau_L)] \]

and conveniently defining the equivalent noise bandwidth of the DLL as

\[ B_L = \frac{2B_s}{N} \frac{1}{2\pi j} \oint_{|z|=1} H_N(z) H_N(z^{-1}) z^{-1} dz \]

this yields to

\[
\sigma^2_\phi = \frac{N_W B_L}{P}
\]

which is identical with the analog result.

The evaluation of the integral \( I = \frac{1}{2\pi j} \oint_{|z|=1} H_N(z) H_N(z^{-1}) z^{-1} dz \) can be evaluated by means of the residual distribution technique. For a 1st order loop it is possible to obtain
\[ H_N(z) = \frac{G_1}{z + (G_1 - 1)} \]
\[ B_L = \frac{2B_s}{N} \frac{G_1}{2 - G_1} \]  
(12.13)

and for a 2nd order loop,

\[ H_N(z) = \frac{(G_1 + G_2)z - G_1}{z^2 + [(G_1 + G_2) - 2]z + (1 - G_1)} \]
\[ B_L = \frac{2B_s}{N} \frac{2G_2 + G_1G_2 + 2G_1^2}{G_1|4 - G_2 - 2G_1|} \]
Chapter 13

Tracking of the Galileo BOC(1,1) modulation

This chapter deals with the tracking performance of the coherent delay lock loop when is applied to the Galileo BOC(1,1). The simulation results will be compared with the analytical model analyzed in Chapter 12 for different discriminator spacings and loop bandwidths.

13.1 Tracking systems for Galileo BOC(1,1) signal and comparison with the GPS system

Chapter 11 has already pointed out the differences between the Galileo BOC(1,1) modulation and the GPS C/A code. The presence of the secondary code does not affect the tracking architecture since the acquisition stage can recover the starting point of both the primary and secondary codes. One peculiarity of the Galileo tracking block is the possibility to use the pilot channel and to extend the coherent integration time, theoretically, without any constraint.

However, the presence of the side peaks in the Galileo BOC modulations may lead the tracking block to erroneously lock on a secondary peak affecting the range measurements with a non negligible bias. This effect can be taken into account both at the acquisition level, minimizing the probability to acquire a side lobes, or at the tracking level. Reference [48] and [49] deal with the problem of the side lobes at the tracking level. In this thesis this problem will not be taken into account, since it has been proved in Chapter 11 how the local maximum technique can be successfully employed to estimate the right main peak delay. Therefore, the tracking stage is initialized in its pull-in range without a possible code delay ambiguity.
13.2 Simulation parameters

The simulation parameters are based on the analysis of Chapter 7. The input signal is a baseband signal whose center frequency is

\[ f_{\text{BB}} = 8 \text{ kHz} \quad \text{(13.1)} \]

The sampling rate is set on the basis of the discussion of Chapters 6 and 7 and it is equal to

\[ f_s = 2.0578 \times 2 \times R_{\text{BOC}} = 2.0578 \times 2 \times 1.023 \text{ MHz} = 4.210256 \text{ MHz} \quad \text{(13.2)} \]

As addressed in Chapter 12 the correlator spacing adopted is lower than \( \frac{1}{3} \) since many discrimination functions require such a value to work properly.

The integration loop time is 20 ms which corresponds to 5 Galileo primary code periods and the equivalent noise DLL bandwidth is in the range

\[ B_L = [1 - 10 \text{ Hz}] \]

13.3 Equivalent noise bandwidth comparison

The performance of the variance of the code phase error, generally addressed as tracking jitter, has been verified by computer simulations with the parameter described in Section 13.2. Each value of the simulation results is obtained based on 60 seconds data (after the code phase acquisition) discarding 10 seconds of transient.

Figures 13.1 and 13.2 show the code tracking jitter comparison between the simulation results, marked values, and the theoretical model of Equation (12.3.3) represented with the solid line.

Figure 13.1 shows the results in the case of a discriminator spacing of 0.25 chips and equivalent DLL noise bandwidth of 10, 5, 2 and 1 Hz respectively, while Figure 13.2 depicts the same situation but when the discriminator spacing is set to the value of 0.125 chip. In order to better quantify this comparison analysis some simulation and theoretical results are reported in Tables 13.1 and 13.2 for different bandwidths and spacing.

The simulations performed prove the reliability of the model developed in Chapter 12, especially for small values of loop bandwidths. In fact, it has to be remarked that the analytical model has been carried out from the linear DLL model, which is a good approximation of the real situation, when the functioning of the DLL can be considered linear. Under the hypothesis of signal lock, for small values of the equivalent DLL noise bandwidth the code phase error \( \phi(z) \) (see Figure 12.5) is kept much closer to the discriminator origin, due to the more noise smoothing effect of the loop filter \( F(z) \), and the
approximation $D(z) = K$ used in the linear DLL model represents a better approximation of the real discriminator behavior.
13 – Tracking of the Galileo BOC(1,1) modulation

**Table 13.1.** Tracking error comparison between simulation and theoretical results expressed in meters, for a discriminator spacing of 0.25 chip

<table>
<thead>
<tr>
<th>CNo</th>
<th>$B_L = 10 \text{ Hz}$</th>
<th>$B_L = 5 \text{ Hz}$</th>
<th>$B_L = 2 \text{ Hz}$</th>
<th>$B_L = 1 \text{ Hz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>4.45</td>
<td>4.17</td>
<td>2.95</td>
<td>2.94</td>
</tr>
<tr>
<td>34</td>
<td>2.81</td>
<td>2.63</td>
<td>1.89</td>
<td>1.86</td>
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<tr>
<td>38</td>
<td>1.78</td>
<td>1.66</td>
<td>1.18</td>
<td>1.17</td>
</tr>
<tr>
<td>42</td>
<td>1.12</td>
<td>1.04</td>
<td>0.75</td>
<td>0.74</td>
</tr>
<tr>
<td>46</td>
<td>0.70</td>
<td>0.66</td>
<td>0.47</td>
<td>0.46</td>
</tr>
<tr>
<td>50</td>
<td>0.44</td>
<td>0.41</td>
<td>0.30</td>
<td>0.29</td>
</tr>
</tbody>
</table>

**Table 13.2.** Tracking error comparison between simulation and theoretical results expressed in meters, for a discriminator spacing of 0.125 chip

<table>
<thead>
<tr>
<th>CNo</th>
<th>$B_L = 10 \text{ Hz}$</th>
<th>$B_L = 5 \text{ Hz}$</th>
<th>$B_L = 2 \text{ Hz}$</th>
<th>$B_L = 1 \text{ Hz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>3.19</td>
<td>2.94</td>
<td>2.16</td>
<td>2.08</td>
</tr>
<tr>
<td>34</td>
<td>2.02</td>
<td>1.86</td>
<td>1.34</td>
<td>1.31</td>
</tr>
<tr>
<td>38</td>
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<td>46</td>
<td>0.49</td>
<td>0.47</td>
<td>0.34</td>
<td>0.33</td>
</tr>
<tr>
<td>50</td>
<td>0.31</td>
<td>0.29</td>
<td>0.21</td>
<td>0.20</td>
</tr>
</tbody>
</table>

13.4 Correlator Spacing Comparison

The analysis of the impact of the discriminator spacing on the tracking performance is highlighted in Figure 13.3. A tracking loop characterized by an equivalent noise bandwidth of 5 Hz and a correlator spacing of 0.125 and 0.25 chip spacing are considered. The solid line refers to the analytical result of Equation (12.3.3) and the marked values to the simulation results.

It is possible to see how the reduction of the correlator spacing leads to a smaller tracking jitter, and then to a better estimate of the range measurements, due to the smaller noise contribution at the discriminator output, according to the consideration pointed out in Chapter 12.
13.5 Step Transient response

The results so far presented show how both the correlators spacing and the equivalent noise loop bandwidth are key parameters in order to achieve the required system performance. However, this analysis just considers the steady state tracking performance without taking into account the system response due to the system dynamics.

In this section the loop response for a 1st order loop system is studied, but the same analysis can be easily extended to higher DLL orders. With \( N_D[n] = 0 \) from Equations (12.3) and (12.4) the following tracking error iterative equation can be derived

\[
\phi[n+1] = [\xi[n+1] - \xi[n]] + [1 - G_1] \phi[n]
\]

The variation of the code phase \( \phi[n+1] \) is the difference between the input dynamic \( \xi[n+1] - \xi[n] \) and the dynamic tracking ability of the loop represented by \( (1 - G_1) \phi[n] \). A reduction of the equivalent noise DLL bandwidth means a reduction of the filter coefficient \( G_1 \) and consequently a more dependence on the system history. Analogously to the analogical counterpart, large values of \( G_1 \) cause the DLL to be sluggish. On the other hand, small values of \( G_1 \) provide much greater attenuation of the input noise. From the above analysis it is possible to conclude that, concerning the transient response, as \( G_1 \) decreases, the loop bandwidth decreases according to Equation (12.13), which may increase the transient duration but decreases the tracking error.

Figure 13.4 shows this situation with some simulation results, where the code phase error \( \phi[n+1] - \phi[n] \) is depicted for decreasing values of the tracking loop bandwidth and
a discriminator spacing of 0.25 chips.

The simulations results agree with the theoretical consideration carried out in this section, in fact it is possible to see how for greater values of the tracking loop bandwidth the steady state is reached in a lower time jointly to a higher error around the steady value. For lower tracking bandwidths the tracking block results to be more robust to the input noise, but the price to be paid is a longer convergence time. Therefore, the choice of the loop tracking parameters plays a very important role and a compromise in terms of noise performance and capability to follow the input signal dynamics has to be achieved.

13.6 Conclusion

In this chapter the conventional algorithm already used for the GPS signal tracking has been investigated, under the assumption of an Acquisition system capable to initialize
the tracking block with the right main peak delay, then avoiding the possibility of a false
lock due to the presence of side lobes on the Galileo BOC(1,1) correlation function.

The impact of tracking parameters such as the discriminator spacing and loop filter
coefficients, on the tracking performance, has been analyzed and the simulation results
compared to the theoretical model developed in Chapter 12, which has been proved to
be a good approximation of the tracking block system.

Finally, the relations between the tracking loop bandwidth and transient response
for a 1st order loop has been studied. A comparison of the step responses for different
loop bandwidths has been performed by means of computer simulations to confirm the
theoretical analysis carried on in this chapter.
Part V

Quality Control
Chapter 14

Quality monitoring techniques

This chapter introduces the concept of Quality Control; the main Quality Control in the field of navigation application and purposes will be addressed with an overview of the state of the art. Particular attention will be pointed out on the signal processing techniques applied to the tracking stage, which will be the main topic of the following chapter of this thesis.

14.1 Quality Control in Navigation applications: Overview of the state of the art

The use of GNSS for safety-of-life applications is increasing rapidly; for such a reason, the measurement accuracy is quite important as well as the integrity monitoring. Moreover in some particular applications as in the case of the aircraft landing approach, the reliability of the solution computed by the navigation receiver process is even more important than a better measurement accuracy.

Galileo will provide integrity information on a global scale through specific elements of the control segment (using also regional components) to users of all services except for Open Service. On the other hand, actual operating GNSS do not provide integrity service directly; for this reason there are commercial differential GPS (DGPS) services. The received signal can be affected by various kind of interference and errors, that can be grouped in:

- **wide band interference**: it is a signal with a constant energy spectrum over all frequency;
- **narrow band interference**: it is a signal with a limited bandwidth, usually of few MHz;
- **evil waveforms (EWF)**: they result from a failure of the signal generating hardware on-board the space vehicle. They cause distortions of the autocorrelation peak.
They are rare events, but in local area differential systems, undetected EWF may cause large pseudorange errors;

- **multipath**: it is a signal distortion due to the presence of multiple paths for the signal.

*Quality control* is the process that defines how well the solution of a problem is known and it consists of assuring an agreed level of accuracy, reliability and robustness for the estimated position. Techniques for evaluating the quality of the estimated position solution can be based on the observation of several different parameters and they can be assessed at different navigation system levels. A well established way to evaluate the quality of positioning is based on the processing of several Position, Velocity and Time (PVT) solutions, at the output of the receivers. For instance, Receiver Autonomous Integrity Monitoring (RAIM) techniques have been developed and refined over the past 10 years to ensure that a given solution is within tolerable constraints. Reference [50] provides a good overview of the literature relating to the significant developments and studies in RAIM techniques. Other possible strategies can be based on Dilution of Precision (DOP) indicators such as BDOPs for baseline relative positioning and ADOPs specifically for ambiguity resolution [51].

A different approach to the quality monitoring can be based on the processing of raw data received such as raw pseudoranges or carrier frequency/phase priori to the solution computation. For example, there has been a large increase in the use of quality control techniques in the context of GPS surveying where they are applied to ensure accurate and reliable survey results [52].

Although they have become an integral part of the GPS Surveying process, the implementation of Quality Control principles inside the tracking loop at the signal processing level, prior to the measurements stage, are not so common. Significant efforts over the past four years have been made to develop and analyze Monitoring Techniques and interference detection strategies based on the analysis and shape of the PRN autocorrelation function. Among others [53] discusses the threats, detection requirements, and detector design approach to mitigate the failures in the WAAS LNAV/VNAV system, while [54] analyzes the latest proposed ground Signal Quality Monitoring (SQM) techniques against several types of failures and *Evil Waveforms* on GPS signals. A multi-correlator scheme for interference monitoring and a metric test based approach for signal validation is presented [55]. More recently [56, 57, 58] propose to use an Extended Kalman Filter based tracking loop for weak and multipath affected GPS signals.

Different quality control methods have been then studied and evaluated; they are based on the observation of different parameters, but they can be grouped mainly in two different classes:

1. techniques based on the processing of PVT solutions;
2. techniques based on the processing of raw data received, prior to PVT calculations.
A simple estimation of the measurement quality is implemented over all the receivers. In fact every receiver calculates the $C/N_0$ ratio and it can be used as a threshold: if the received ratio is under a chosen level, then the measure is considered not reliable. Another kind of estimation is the **Dilution of Precision** (DOP) measurement. The accuracy of the computed location depends on the user/satellite relative geometry. The user will be located in an uncertainty region (shaded region in Figure 14.1) smaller in the case of good geometry than in the bad one: larger is the separation angle between satellites seen from observer position, better is the accuracy reached.

![Figure 14.1. Relative geometry and DOP: on the left (a) geometry with low DOP, and on the right (b) geometry with high DOP](image)

The best geometry with four satellites (the minimum number of required satellites for a PVT solution computation) occurs when three satellites are on the horizon plane and the fourth at the user’s zenith (Figure 14.2).

![Figure 14.2. Best satellites geometry](image)
14 – Quality monitoring techniques

14.2 Methods based on PVT solutions

It is increasingly important in many applications that faulty operations have to be avoided or quickly detected and eliminated [59]. The same importance is given to the low probability of a false alarm. Some autonomous methods for monitoring system integrity will be now described.

14.2.1 RAIM

The Receiver Autonomous Integrity Monitor is a monitoring method internal to the receiver used to check the Position Velocity Time (PVT) solution consistency. External methods can be used too, but the latency introduced in the error detection makes RAIM the preferred methods adopted. RAIM derives from the PVT some statistic tests.

The most common test are:

- **maximum solution separation**: considering $n$ measurements, $n$ solutions are computed by leaving one of the measurements out of each solution. Then all possible distances between these solutions are calculated and the maximum of them is used as a test statistic [60];

- **least square residual method**: the residual from measurements is calculated and then used to calculate some metrics; when these values exceed a threshold value, a fault will be declared and an attempt will be made to determine the measurement and associated satellite at fault [59].

RAIM monitors the system integrity, since it determines the ability of the system to give users the warning that the measurements are reliable or not. Therefore, it is also important since gives a measurement of the system detection power.

14.2.2 FDI and Kalman filtering

Determining which is the satellite is the cause of a detected error would be very useful, since it would be possible to try to isolate it. The measurement, after the satellite isolation, would become reliable again. This particular method is the so called Failure Detection and Isolation (FDI).

Kalman filters (see Chapter 15) could be used to handle measurements which may be error affected. The satellites are never excluded, but a weight is associated with their measurements on the basis of the estimation of the error amount done by the Kalman filter. Since the measurements are never excluded, possible losses of information is then reduced.
14.3 Methods based on signal processing

Another approach to the quality control problem is to consider the raw data received from the satellites and to process them before the navigation computation. An evil waveform causes distortion in the correlation function inside the receiver tracking channel, therefore inducing a different measurement error for two receivers that would not have the same architecture. As a consequence, EWF can potentially induce large tracking errors in differential systems if left undetected [61]. Detection of such distortion is difficult because the ranging error depends on the spread spectrum receiver discriminator type, correlator spacing and bandwidth [53]. This leads to increase the complexity of the receiver.

A Signal Quality Monitoring (SQM) scheme would consist of one or more (wideband) GPS receivers having several correlators configured to sample the correlation peak at multiple location in order to determine its level of distortion. The detection capability of a particular SQM is limited by the nominal distortion of the correlation peak; it is quantified in terms of Minimum Detectable Error (MDE), that is dependent on satellite elevation angle and dominated by multipath conditions. If an EWF is detected, the corresponding satellite would be flagged and its pseudorange removed from PVT solution [62]. Test metrics are generated using multiple correlators outputs; the mainly used are based on the following ratio [61]:

\[
\Delta_{\pm d} = \frac{I_{-d} - I_{+d}}{2I_p}
\]

\[
R_{\pm d} = \frac{I_{-d} + I_{+d}}{2I_p}
\]

\[
R_d = \frac{I_d}{I_p}
\]

where

- \( d \) is the offset from the prompt code in chips;
- \( I_d \) is the output of the \( d \) correlator on the I channel;
- \( I_p \) is the output of the I prompt correlator.

The test metrics calculated using these ratios are then compared with the MDE threshold. MDE are computed so that both the false alarm rate and the probability of missed detection of a misleading information (that is an undetected pseudorange differential error greater than the maximum error that can be tolerated) are as low as possible. Multiple DLL have been used too, each of them having a different chip spacing: the differences between measurements from the first and the second DLL and from second and third are compared to a test threshold.
All the techniques described so far are receiver–based; in reference [63], a satellite–based integrity monitor is presented. Difficulty and highly cost operations needed to operate ground–based integrity monitoring, such as in Wide Area Augmentation System (WAAS), would be greatly lessened if integrity monitoring would be conducted within the satellite constellation itself. Ground–based stations take several seconds to alert users after the discovering of a failure; in addition, also ground control stations of the GPS system take too time to alert users (in the order of minutes). In safety critical applications, in which the real–time alert capability is a requirement, this technique is not affordable. Satellite Autonomous Integrity Monitoring (SAIM) can resolve the problem of rapid failure alerting by integrating detection and alerting within the satellite itself. Once a failure is detected, a message is sent to the signal processor to change the navigation message such as it is immediately unusable.

14.3.1 Quality monitoring at the signal tracking level based on Extended Kalman Filter

In a navigation receiver the code tracking block tries to maximize the cross-correlation between the local generated code and the received signal, on the basis of the autocorrelation function of the PRN codes. In fact, when the codes are perfectly aligned the auto-correlation assumes the maximum value. Lock of the signal might be maintained by feeding back a proper control signal which regulates the local code phase.

When multipath is present, it changes the cross-correlation function used for the alignment in the tracking stage. There are several techniques to mitigate the multipath effect, such as the Narrow correlator, Edge and Strobe correlators. None of these methods give information about how much the cross-correlation function departs from the triangular shape as a fault consequence.

As already mentioned before, Kalman filter has been inserted in the tracking loop in order to increase the system performance of the DLL. By the way, none of these techniques have been concentrated on the aim of quality monitoring the shape of the autocorrelation function, and therefore provide the information on the reliability of the tracked channel. This technique will be deeply investigated in the Chapter 16 where a modified tracking architecture is presented and theoretically analyzed.
Chapter 15

Kalman filtering

The Kalman filtering is a technique able to estimate the values of the system states, in an optimal way under the hypothesis of Gaussian noise. It is a way to solve the minimum mean square error filtering problem using state space methods. It is a set of mathematical equations that provide an efficient computational means to estimate the state of a process, in a way that minimizes the mean of the squared error. The filter is a very powerful tool and it can estimate past and future states even when the precise nature of the modeled system is unknown.

15.1 Mathematical description

The first step in the design of a Kalman filter is to model the system and the measurement noise in a vector form. It is generally assumed that the random process to be estimated can be written in the form \[ x_{k+1} = \phi_k x_k + w_k \] (15.1) and that it is possible to model the observations as \[ z_k = H_k x_k + v_k \] (15.2)

where

- \( x_k \) is the process state vector
- \( \phi_k \) is the state transition matrix
- \( w_k \) is the process noise vector, assumed to be a white sequence
- \( z_k \) is the measurement vector
$H_k$ is the connection matrix between the measurement and the state vector
$v_k$ is the measurement noise vector, assumed to be a white sequence

The noise vectors, $w_k$ and $v_k$, have a known covariance structure and zero cross-correlation between each other; in formulas:

$$E[w_k w_i^T] = \begin{cases} Q_k & i = k \\ 0 & i \neq k \end{cases}$$

(15.3)

$$E[v_k v_i^T] = \begin{cases} R_k & i = k \\ 0 & i \neq k \end{cases}$$

(15.4)

$$E[w_k v_i^T] = 0 \quad \forall i, k$$

(15.5)

It is assumed to know the so called “a priori state estimate”, which is an initial estimate of the process $\hat{x}_k$ and the initial error covariance matrix associated to $\hat{x}_k^\rightarrow$. The estimation error is

$$e_k^\rightarrow = x_k - \hat{x}_k^\rightarrow$$

(15.6)

and the associated error covariance matrix is

$$P_k^\rightarrow = E[e_k^\rightarrow e_k^\rightarrow^T] = E[(x_k - \hat{x}_k^\rightarrow)(x_k - \hat{x}_k^\rightarrow)^T]$$

(15.7)

The measurements at time $t_k$ are then used to improve the a priori estimate

$$\hat{x}_k = \hat{x}_k^\rightarrow + K_k (z_k - H_k \hat{x}_k^\rightarrow)$$

(15.8)

where

$\hat{x}_k$ is the updated estimate

$K_k$ is the filter gain

The filter gain $K_k$ is obtained minimizing the mean square error of the estimate to provide an optimal update. The covariance matrix associated with the updated (a posteriori) estimate [64] is then formed.

$$P_k = E[e_k e_k^T] = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T]$$

(15.9)

Substituting Equation (15.2) into Equation (15.8) and writing the new expression of $\hat{x}_k$ into Equation (15.9), it follows

$$P_k = P_k^\rightarrow - P_k^\rightarrow H_k^T K_k^T - H_k K_k P_k^\rightarrow + K_k (H_k P_k^\rightarrow H_k^T + R_k) K_k^T$$

(15.10)
The estimate mean square errors to be minimized are located along the major diagonal of $P_k$, since they represent the estimation error variances. This corresponds to minimize the trace of $P_k$, because it is the sum of the mean square errors in the estimates of the process vector. Starting from

$$\frac{d \left( \text{trace} \left( P_k \right) \right)}{d K_k} = 0$$

it is possible to obtain

$$K_k = P_k^{-1} H_k^T \left( H_k P_k^{-1} H_k^T + R_k \right)^{-1} \quad (15.11)$$

The so calculated gain takes the name of Kalman gain [64]. Several expressions of the updated covariance matrix can be derived with the substitution of the optimal gain into Equation (15.10).

Equation (15.12) will be adopted in the following

$$P_k = (I - K_k H_k) P_k^{-} \quad (15.12)$$

In any recursive procedure the current step is used to obtain the desired result for the successive step. In order to make an optimal use of measurements at $t_{k+1}$, the Kalman filter needs to know $\hat{x}_{k+1}^{-}$ and $P_{k+1}^{-}$. These quantities can be obtained projecting ahead the updated estimated $\hat{x}_k$ through the process state transition matrix

$$\hat{x}_{k+1}^{-} = \Phi_k \hat{x}_k \quad (15.13)$$

neglecting the contribution of $w_k$, since it is an uncorrelated zero mean process. All the pieces to write the expression for the a priori error are then available

$$e_{k+1}^{-} = x_{k+1} - \hat{x}_{k+1}^{-} = (\Phi_k x_k + w_k) - \Phi_k \hat{x}_k = \Phi_k e_k + w_k \quad (15.14)$$

Finally, the expression for the a priori covariance error matrix at $t_{k+1}$ can be easily obtained

$$P_{k+1}^{-} = E \left[ e_{k+1}^{-} e_{k+1}^{-T} \right] = \Phi_k P_k \Phi_k^T + Q_k \quad (15.15)$$

The Kalman filter recursive equations are expressed by Equation (15.8), Equation (15.11), Equation (15.12) and Equation (15.13). The filter loop can be continued infinitely or it can be chosen to stop it, for example when the measurements are over.

The basic steps of the algorithm are

161
1. enter prior estimate \( x_k^- \) and its error covariance matrix \( P_k^- \)

2. computation of the Kalman gain: 
   \[ K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \]

3. update estimate with measurements: 
   \[ \hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-) \]

4. compute error covariance matrix for updated estimate: 
   \[ P_k = (I - K_k H_k) P_k^- \]

5. project ahead: 
   \[ \hat{x}_{k+1}^- = \Phi_k \hat{x}_k \quad \text{and} \quad P_{k+1}^- = \Phi_k P_k \Phi_k^T + Q_k \]

6. again from step 2

### 15.2 Linearization

All the considerations addressed in the previous section can be used in the design of a Kalman filter to be used in a linear system. Unfortunately real systems are generally non linear with or without non-linear measurements.

The Kalman filter can also be used in these cases, but these nonlinearities have to be linearized. Two methods are possible: linearize about on a nominal trajectory in the state space which is not depend on measurements or about a trajectory that is updated with the state estimates resulting from the measurements \cite{64}. A trajectory is a particular solution of a stochastic system \cite{65}, obtained with particular values of the random variables involved.

#### 15.2.1 Linearized Kalman filter

Denoting with \( x^* \) the nominal trajectory the actual trajectory can be written as

\[ x = x^* + \Delta x \quad (15.16) \]

The measurements relationship and the process to be estimated can be written as

\[ \dot{x}^* + \Delta \dot{x} = f(x^* + \Delta x) + u \quad (15.17) \]
\[ z = h(x^* + \Delta x) + v \quad (15.18) \]

where \( f \) and \( h \) are nonlinear functions. Equations (15.17) and (15.18) can be linearized by means of the Taylor’s series expansions. Assuming \( \Delta x \to 0 \) and considering just the first–order terms \cite{64}

\[ \dot{x}^* + \Delta \dot{x} \approx f(x^*) + \left[ \frac{\partial f}{\partial x} \right]_{x=x^*} \cdot \Delta x + u \quad (15.19) \]
\[ z \approx h(x^*) + \left[ \frac{\partial h}{\partial x} \right]_{x=x^*} \cdot \Delta x + v \quad (15.20) \]
where

\[
\frac{\partial f}{\partial \mathbf{x}} = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots \\
\vdots & \vdots & \ddots \\
\end{bmatrix}, \quad \frac{\partial h}{\partial \mathbf{x}} = \begin{bmatrix}
\frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots \\
\frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \cdots \\
\vdots & \vdots & \ddots \\
\end{bmatrix}
\]

(15.21)

The nominal trajectory is chosen to satisfy the deterministic differential equation

\[
\dot{\mathbf{x}}^* = f(\mathbf{x}^*)
\]

(15.22)

The linearized model can be finally obtained substituting Equation (15.22) into Equation (15.17)

\[
\Delta \dot{\mathbf{x}} = \left[ \frac{\partial f}{\partial \mathbf{x}} \right]_{\mathbf{x}=\mathbf{x}^*} \cdot \Delta \mathbf{x} + \mathbf{u}
\]

(15.23)

\[
[\mathbf{z} - h(\mathbf{x}^*)] = \left[ \frac{\partial h}{\partial \mathbf{x}} \right]_{\mathbf{x}=\mathbf{x}^*} \cdot \Delta \mathbf{x} + \mathbf{v}
\]

(15.24)

This strategy is generally addressed as Linearized Kalman Filter.

### 15.2.2 Extended Kalman Filter (EKF)

The extended Kalman filter differs from the linearized Kalman filter since the partial derivatives of Equation (15.21) are evaluated along a trajectory that has been updated with the filter estimates. The selected trajectory just depends on the measurements [64].

For the case of the EKF, it is more convenient to consider total quantities of the estimates. The incremental estimate update equation is

\[
\Delta \hat{\mathbf{x}}_k = \Delta \hat{\mathbf{x}}_k^- + \mathbf{K}_k \left[ \mathbf{z}_k - h(\mathbf{x}_k^*) - \mathbf{H}_k \Delta \hat{\mathbf{x}}_k^- \right]
\]

(15.25)

where the expression of Equation (15.25) written between brackets is the measurement residual, which can be written as

\[
\mathbf{z}_k - \hat{\mathbf{z}}_k^-
\]

(15.26)

and

\[
\hat{\mathbf{z}}_k^- = h(\mathbf{x}_k^*) + \mathbf{H}_k \Delta \hat{\mathbf{x}}_k^-
\]

In this way, the measurement residual (Equation (15.26)) is formed exactly as expected from the EKF, that is, it is the noisy measurement minus the predictive measurement based on the corrected trajectory rather than the nominal one [64]. Adding \( \mathbf{x}_k^* \) to both sides of Equation (15.25) the linear estimate update equation written in terms of total
quantities is obtained; the a priori estimate is corrected by adding the measurement residual weighted by the Kalman gain.

\[ \hat{x}_k = \hat{x}_k^- + K_k [z_k - \hat{z}_k^-] \]  
(15.27)

\( \hat{x}_k \) is projected ahead through nonlinear dynamics

\[ \dot{x} = f(x) + u \]

Once \( \hat{x}_{k+1}^- \) is determined, the predictive measurement \( \hat{z}_{k+1}^- \) can be formed as \( h(\hat{x}_{k+1}^-) \).

The measurement residual at \( t_{k+1} \), besides, is formed as the difference \( (z_{k+1} - \hat{z}_{k+1}^-) \).
In Chapter 14 it has been described the state–of–the art of Quality Monitoring and Control techniques for GNSS. Two main classes have been introduced: the first based on PVT calculations and the second based on signal processing. In this chapter a Quality Control technique based on signal processing techniques is introduced; a modified tracking architecture, where an Extended Kalman Filter is inserted, will be studied and its performance evaluated by means of computer simulations.

16.1 DLL modified architecture

One of the error sources for a GNSS receiver is the presence of multipath rays (see Section 4.6): they are reflections of the Line Of Sight (LOS) ray from satellite to user, caused by the presence of obstacles, such as buildings and trees, or from ground reflections. Multipath (MP) signals are delayed with respect to the LOS signal because of the different paths they travel over. Furthermore, the reflection is generally source of energy dissipation, so the MP has generally a lower power contribution than the direct signal. It is here recalled how, at the receiver, the signal can be considered equal to the sum of the LOS and MP signals (Chapter 4.6):

\[ s_R(t) = s(t) + \sum_{k=1}^{N} m_k s(t - \tau_k) \]  

(16.1)

where

\( s_R(t) \) is the received signal;

\( s(t) \) is the transmitted signal;
$m_k$ is the amplitude of the $k$–th MP ray;

$\tau_k$ is the delay of the $k$–th MP ray.

The effect of the multipath is a distortion of the autocorrelation function, as it can be seen in Figure 16.1 and the consequence is a tracking error, which affects the navigation solution and then its precision.

![Figure 16.1. Effect of multipath on the correlation function](image)

The conventional DLL uses discriminator functions constructed with a specific combination of its early, prompt and late correlators to detect code tracking error. It is well known that this architecture would suffer from performance degradation due to error sources like multipath, interference, momentary loss of signal or weak signals received from the side lobe of the antenna. A possible extension of the conventional DLL architecture bringing the idea of the quality on the tracking measurement consists of multiple correlators and an opportune Kalman filter with an appropriate loop filter. The Kalman filter extracts and estimates code tracking error of the direct satellite signal component from the corrupted input signals by averaging multiple samples of the PRN code autocorrelation function. On the basis of an opportune stochastic model the Kalman filter can be used to evaluate the multipath components which can be pulled out from the direct signal, mitigate the loss gain in the discrimination function and correlation blocks in the presence of a weak signal and predict the system evolution of the incoming signal even in a momentary loss of the signal itself. The insertion of a Kalman filter inside the tracking block can also be used to estimate the reliability of the incoming signal by comparing the measurement results with an opportune cost function and, where possible, to mitigate the influence of temporary interferences.

Figure 16.2 shows the architecture of the modified DLL, it is possible to see how the correlator branches are also the input of Kalman filter and used to build the discrimination function and in the normal loop operations. In the following sections the modified
tracking architecture is analyzed considering a channel affected by a single multipath component. The estimation of the amplitude and delay of the reflection make possible to evaluate the error on the measurements and then to deduce the current quality of the signal tracked.

Increasing the number of correlator branches in the DLL, it is possible to feed the Kalman filter with a higher number of measurements and then to achieve generally better performances due to the higher amount of information which can be processed. The price to be paid is of course a bigger computational complexity. Figure 16.3 depicts the reconstruction of the correlation function at the increase of the correlator branches; the case of 7 and 21 correlators is represented and compared with the theoretical correlation function shape.

It is clear that the solution with 21 correlators represents in a better way the theoretical correlation.

### 16.2 Signal model

In Section 3.3 the modulations used in every Galileo frequency band have been described. The L1 channel signals are multiplexed into the L1 carrier using CASM modulation. The L1–B,C signal is modulated on to the carrier in-phase component whilst the L1–A signal...
The received signal can be written as

\[ s_{\text{rx} \ L1-C} (t) = A \cdot s_{\text{rx} \ L1-C} (t - \tau_{\text{L1}}) \cos [2\pi (f_{\text{L1}} - \Delta f_{\text{L1}}) (t - \tau_{\text{L1}}) + \phi] \]  

(16.2)

where \( A \) is the generic signal amplitude and \( s_{\text{rx} \ L1-C} (t) = c_{\text{L1-C}} (t) sc_{\text{L1-C}} (t) \) is the product of the L1 PRN code sequence \( c_{\text{L1}} (t) \) and the sin–phased BOC(1,1) sub–carrier \( sc_{\text{L1}} (t) \).

The PRN code sequence can be written as

\[ c_{\text{L1-C}} (t) = \sum_{m=-\infty}^{+\infty} \sum_{k=0}^{L_{\text{cod}}-1} c_k P_T_c (t - kT_c - mT_{\text{cod}}) \]  

(16.3)

where \( L_{\text{cod}} = 4092 \) chips, \( \tau_{\text{cod}} = 4 \) ms and \( T_c = \frac{T_{\text{cod}}}{L_{\text{cod}}} \), and taking into account the effect of the BOC(1,1) sub–carrier on the ranging code sequence, and without any loss of generality considering a normalized signal amplitude (e.g. \( A = 1 \)), the Galileo BOC(1,1)
16.3 – Kalman filter parameters

The parameters of interest are $A_{\text{LOS}}$, $\tau_{\text{LOS}}$, $A_{\text{MP}}$ and $\tau_{\text{MP}}$, which are used to define the following states vector to be processed by the Kalman filter:

$$\mathbf{x} = [A_{\text{LOS}} \quad \tau_{\text{LOS}} \quad A_{\text{MP}} \quad \tau_{\text{MP}}]^T$$

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In this thesis just the problem of a fixed and single multipath component is taken into account, then the state transition matrix associated to this particular case is simply

\[ \Phi = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (16.10) \]

The measurements \( z \) available to the filter are the I and Q correlation samples, which corresponds the following measurements vector

\[ z = [I_{\delta_1}, I_{\delta_2}, \cdots, I_{\delta_j}, Q_{\delta_1}, Q_{\delta_2}, \cdots, Q_{\delta_j}] \quad (16.11) \]

where \( j \) is the number of correlator branches. Including the MP component the I and Q samples can be written, analogously to the case of Equations (16.6) and (16.7), as

\[
I_{\delta_j} \approx A_{\text{LOS}} R_{\text{BOC}} (\tau_{\text{LOS}} + \delta_j) \cos (\phi_{\text{LOS}}) D_N [\pi \Delta F_D] \\
+ A_{\text{LOS}} R_{\text{BOC}} (\tau_{\text{LOS}} + \tau_{\text{MP}} + \delta_j) \cos (\phi_{\text{MP}}) D_N [\pi \Delta F_D] + n_I
\]

\[
Q_{\delta_j} \approx A_{\text{LOS}} R_{\text{BOC}} (\tau_{\text{LOS}} + \delta_j) \sin (\phi_{\text{LOS}}) D_N [\pi \Delta F_D] \\
+ A_{\text{LOS}} R_{\text{BOC}} (\tau_{\text{LOS}} + \tau_{\text{MP}} + \delta_j) \sin (\phi_{\text{MP}}) D_N [\pi \Delta F_D] + n_Q
\]

The dynamic of the filter parameters cannot be assumed linear, since the correlator measurements are non–linear in the signal parameters. Therefore, an Extended Kalman Filter (see Section 15.2.2) has to be employed.

As written in Section 15.2.2, the EKF needs the linearization of the measurement matrix \( H \), calculating the derivatives of Equation (16.12) with respect to the component of the state vector (here only the I channel derivatives have been evaluated; the only difference between I and Q channel is the presence of the \( \sin \) function in the I channel and of the \( \cos \) function in the Q channel, neither of them affected by the derivative operator):

\[
\frac{\partial I_{\delta_j}}{\partial A_{\text{LOS}}} \approx R_{\text{BOC}} (\tau_{\text{LOS}} + \delta_j) \cos (\phi_{\text{LOS}}) D_N [\pi \Delta F_D] \quad (16.12)
\]

\[
\frac{\partial I_{\delta_j}}{\partial \tau_{\text{LOS}}} \approx A_{\text{LOS}} \frac{d}{d\tau_{\text{LOS}}} \{ R_{\text{BOC}} (\tau_{\text{LOS}} + \delta_j) \} \cos (\phi_{\text{LOS}}) D_N [\pi \Delta F_D] \\
+ A_{\text{MP}} \frac{d}{d\tau_{\text{MP}}} \{ R_{\text{BOC}} (\tau_{\text{LOS}} + \tau_{\text{MP}} + \delta_j) \} \cos (\phi_{\text{MP}}) D_N [\pi \Delta F_D] \quad (16.13)
\]

\[
\frac{\partial I_{\delta_j}}{\partial A_{\text{MP}}} \approx R_{\text{BOC}} (\tau_{\text{LOS}} + \tau_{\text{MP}} + \delta_j) \cos (\phi_{\text{MP}}) D_N [\pi \Delta F_D] \\
+ A_{\text{MP}} \frac{d}{d\tau_{\text{MP}}} \{ R_{\text{BOC}} (\tau_{\text{LOS}} + \tau_{\text{MP}} + \delta_j) \} \cos (\phi_{\text{MP}}) D_N [\pi \Delta F_D] \quad (16.14)
\]

\[
\frac{\partial I_{\delta_j}}{\partial \tau_{\text{MP}}} \approx A_{\text{MP}} \frac{d}{d\tau_{\text{MP}}} \{ R_{\text{BOC}} (\tau_{\text{LOS}} + \tau_{\text{MP}} + \delta_j) \} \cos (\phi_{\text{MP}}) D_N [\pi \Delta F_D] \quad (16.15)
\]
In any system model, there is a connection between states and measurements, but in general it is not possible to directly derive the value of the states from the measurements. Besides, the measurements are commonly noise corrupted. In general the model which describes the states evolution over time is an approximation of the real one and just some parameters of Equation (15.1) are known. Sometimes the system is too complex to be represented and a simplified model is used to represent its evolution. Anyway, this leads to a common problem called tuning problem. In this context tuning means to select the appropriate covariance matrix \( P \), the process noise covariance matrix \( Q \) and the measurement noise covariance matrix which best approximates all the states or just the one under analysis. The matrices tuning is generally done considering the simple case of diagonal matrices, which corresponds to the hypothesis of independence among the system states. In this particular situation this solution seems to be reliable, since any particular dependence is assumed to affect the amplitude and delay of both the LOS and MP components.

Tuning the matrices is generally quite complex and the stability and convergence of the Kalman filter are quite affected by it. This task is extremely important and due to the complexity to solve it in an analytical way is generally done in an empirical way or by means of computer simulations. This second strategy will be followed in this thesis.

Since both the multipath and the line-of-sight signals are supposed to be constant during the time of analysis, the \( Q \) matrix associated to this problem, from Equation (15.1), should be the null matrix. Unfortunately, such a choice makes the filter unstable. In order to study the impact of the \( Q \) matrix on the system stability several simulations have been performed. The case of a MP component affecting the incoming signal, with a relative signal power of a quarter the line-of-sight power and a delay \( \tau_{\text{MP}} \) (relative to the LOS delay) of 0.4 chip is considered as reference case. The diagonal items of the \( Q \) matrix have been increased simulation by simulation from the starting value of \( 10^{-5} \) to reach the value of \( 10^{-2} \) (the choice of these values has been done in an empirical way) for a CNo = 40 dB–Hz and a single integration period. Finally, the simulations have been done feeding the Kalman filters with the output of 15 correlators.

Figure 16.4 shows the case of the LOS and MP amplitude and delay estimation when the \( Q \) matrix is

\[
Q = \begin{bmatrix}
10^{-5} & 0 & 0 & 0 \\
0 & 10^{-5} & 0 & 0 \\
0 & 0 & 10^{-5} & 0 \\
0 & 0 & 0 & 10^{-5}
\end{bmatrix} \tag{16.16}
\]

The simulation of Figure 16.4 shows how this situation wrongly leads to an incorrect multipath estimation. This effect can be attributed to the numerical stability of the procedure how the Kalman filter gain is calculated. The items of the \( Q \) matrix are too small and this leads to a numerical instability of the matrix inversion involved in the Kalman
Figure 16.4. Estimated states with Q values of $10^{-5}$

algorithm.

Figure 16.5 shows the same simulation but with a Q matrix with diagonal items equal to $10^{-4}$.

In this situation the filter becomes able to estimate the amplitude and delay of both the signal and multipath components. However, this is achieved with a long convergence time. In fact, the filter tries to estimate the best state evolution according to the state propagation rule of Equation (15.1). Therefore, extremely low values of the Q matrix

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Figure 16.5. Estimated states with Q values of $10^{-4}$

items may lead the filter to be less reactive to achieve the steady state.

Figures 16.6 and 16.7 show the estimations when the Q matrix items are increased progressively from $10^{-3}$ to $10^{-2}$.

It is possible to see how a better estimation in a lower convergence time can be achieved with bigger Q matrices. The only drawback is an increased ripple of the estimates around the true values. One of the targets of the tuning problem is to achieve a trade–off between achievable convergence and accuracy on the estimates.
In order to analyze the accuracy of the estimation capability of the modified architecture, in Figure 16.8 and 16.9 the LOS and MP amplitude and delay estimations are reported associated to the 3–σ confidence interval. The situation refers to the choice of a $Q$ with diagonal items equal to $10^{-2}$.

It has to be highlighted, how the choice of a diagonal $Q$ matrix with items all equal
16.3 – Kalman filter parameters

Figure 16.7. Estimated states with Q values of $10^{-2}$

...leads to a more robust filter implementation in terms of stability and a lower convergence time to achieve the estimation. By the way, even though it produces a good parameters estimation the $3-\sigma$ confidence intervals may be too wide in many applications. This problem can be overcome relaxing the choice of having a Q matrix with the same diagonal items. This is the case of Figure 16.10 and 16.11 that report the LOS and MP amplitude and delay estimations, with the associated $3-\sigma$ confidence intervals, with an "ad hoc" selection of the Q matrix items.
The results prove that even a better convergence time and a better estimation can be achieved. The price to be paid is a more difficult filter tuning, and the performance associated to a specific choice of the Q matrix may be scenario dependent (i.e., from the multipath amplitude and delay, carrier–to–noise ratio, integration time interval, number...
of correlators employed and so on). This is a limiting factor of this techniques especially for a real implementation on a mass-market receiver.

In order to better appreciate the estimation capabilities of the architecture of Figure 16.2, the main estimation statistics, discarding in their evaluation the convergence.
required by the Kalman filter, are summarized in Table 16.1. In the same table, the extreme limits of the $3-\sigma$ confidence intervals are stated together with the average and variance of the estimation error defined as the difference between the real and estimated values.

<table>
<thead>
<tr>
<th></th>
<th>Mean Value</th>
<th>Average error</th>
<th>Error variance</th>
<th>Confidence limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Upper</td>
</tr>
<tr>
<td>LOS Amplitude</td>
<td>0.99</td>
<td>$1 \cdot 10^{-2}$</td>
<td>$2.22 \cdot 10^{-3}$</td>
<td>1.109</td>
</tr>
<tr>
<td>LOS Delay</td>
<td>$-4.7 \cdot 10^{-2}$</td>
<td>$4.7 \cdot 10^{-2}$</td>
<td>$1.3 \cdot 10^{-3}$</td>
<td>$5.3 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>MP Amplitude</td>
<td>0.53</td>
<td>$-3 \cdot 10^{-2}$</td>
<td>$1.9 \cdot 10^{-3}$</td>
<td>0.63</td>
</tr>
<tr>
<td>MP Delay</td>
<td>0.39</td>
<td>$1 \cdot 10^{-3}$</td>
<td>$5.0 \cdot 10^{-4}$</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 16.1. Estimation results and statistics for an "ad hoc" tuning of the $Q$ matrix

16.4 Conclusion

In this chapter a modified tracking architecture with Quality Control features has been studied. The performance in terms of capability of LOS and multipath amplitude and delay estimation have been analyzed. The situation which is considered is the simple environment where a single and fixed multipath affects the line of sight signal. The analysis performed has shown how the modified architecture can be used to estimate the multipath parameters, such as its amplitude and delay, in a noisy environment using the correlator outputs as input of a Kalman filter. The modified architecture presents the so-called tuning problem: the system parameters have to be selected with particular care in order to avoid possible convergence problems. A trade–off in terms of system reactivity and accuracy is then a critical design parameter. In this work, this problem due to its complexity has been solved in an empirical way by modifying the items of the $Q$ matrix assumed to be diagonal.

The Kalman filter is able to estimate the input signal parameters, while the signal is tracked by the DLL. This feature is extremely interesting since it is possible to associate, to the current tracked signal a measure of its quality.

Three are the main drawbacks of this DLL structure: the problem regarding the tuning of the operational parameters and the filter stability, the input signal model definition and the computational complexity of the architecture. In order to achieve good performance, the Kalman filter has to be "trained" with the knowledge of which kind of input has to deal with and its stochastic nature, but unfortunately this information are not always available. Moreover, even when this information are known or partially available, it is not generally easy to represent the states dynamic as in the form of Equation (15.1) without increasing the number of states to be estimated and consequently increasing
the computational requirements. However, the possibility to use a Kalman filter in the tracking loop looks like very attractive as demonstrated in the simulation results shown in this chapter.
Conclusions

The purpose of this thesis was to investigate the impact of the Galileo L1F signal on a pure software based navigation receiver. The results obtained so far show that the software radio approach can be feasible applied to develop a fully navigation software receiver on a reconfigurable hardware platform.

This work was mainly subdivided in the analysis of the acquisition techniques able to acquire the new Galileo signal broadcast on the L1F frequency, the investigation of the impact of such a signal on the conventional tracking blocks and finally the study of a novel tracking architecture with the capability to assess the quality of the signal tracked in terms of multipath estimation. For this reasons, from this preliminary activity, recommendations may be made for an operational receiver as well as for future work. In the following a list of activities and problems that emerge working toward a practical implementation has been outlined.

1. Dealing with a digital receiver means to work with a sequence of samples. If the sampling frequency is not properly selected with respect to the PRN code rate the information of the code transition required for the pseudorange computation may be lost. Same accuracy in terms of pseudorange resolution can be achieved with different values of the sampling frequency and then with a different impact on the instantaneous computational performance required by the platform. The sampling frequency then becomes a key parameter in order to achieve the best performance using the lowest computational hardware requirements.

2. According to the Galileo SIS documentation, two main characteristics of this new signal were studied: the availability of a pilot channel in a quadrature with a traditional data channel and the use of the BOC square sub–carrier. This reduces the possibility to use the conventional acquisition schemes already adopted for GPS. For the fast acquisition scheme the possibility to use a linear correlation, using longer FFTs, has to be taken into account, since during the block of processed data a sign reversal due to a secondary code may occur.
3. To improve the signal to noise ratio a non–coherent summation strategy can be used, both for the serial and parallel acquisition techniques, to overcome again the problem of the secondary code transitions.

4. The use of the BOC modulation introduces side lobes on the autocorrelation function. If they are not taken into account neither at the acquisition nor at the tracking level the receiver can be erroneously lock on one of them with a consequent bias in the pseudorange measurement. In this work a strategy ables to reduce the side lobe detection probability has been proposed for the BOC(1,1) modulation.

5. The digital models of both the acquisition and tracking schemes for a navigation receiver have been developed and proved by means of computer simulations. This study has taken into account the possible losses of the non–idealities which can incur in a real digital implementation. The availability of reliable models is of extremely importance, in the development phase of the software receiver, to understand the best trade–off between the system performance and the computational complexity.

6. Quality Monitoring of disturbance on signals is becoming a growing area of interest for research and commercial development. In this framework a modified tracking architecture ables to estimate the amplitude and delay of a Multipath affecting the received signal is proposed and studied. The estimator based on an Extended Kalman Filter gives the possibility to the tracking block to measure the reliability of the current signal tracked.

The results addressed in this work show the possibility to develop a software Galileo receiver for the BOC(1,1) modulation achieving the operational performance required by a traditional navigation receiver.

Even though this thesis just deals with the L1F signal, it has to be taken into account the possibility to extend all the algorithms herein analyzed to the other signals which will be offered by the future Galileo satellite system and it is possible combination and integration with the actual GPS system and its future modernization.

Finally, although the models and simulation tools developed in this thesis were designed to represent as close as possible the real situation, it would be really valuable to test the different acquisition and tracking strategies on real data. The first Galileo satellite was launched the 28th of December 2005, which is a good opportunity to validate the algorithms investigated in this Thesis.
Part VI

Appendix and Bibliography
Appendix A

Approximated BOC(1,1) autocorrelation function

A simplified expression for the Galileo BOC(1,1) autocorrelation function, valid in the range $T_c \in [\pm 2\text{ chip}]$ can be easily evaluated considering just a single PRN chip modulated by the sub-carrier, with normalized unity energy, as shown in Figure A.1.

![Figure A.1. Single Galileo PRN code modulated by the BOC(1,1) sub-carrier](image)

Since the autocorrelation function of a signal $x(t)$ is defined as the inverse transform of $|X(f)|^2$, in other words

$$R_x(\tau) = \int_{-\infty}^{\infty} |X(f)|^2 e^{j2\pi f \tau} d\tau$$  \hspace{1cm} (A.1)

the approximated expression for the case of a BOC(1,1) signal can be derived calculating the Fourier transform of the the signal in Figure A.1 and by applying the definition of autocorrelation
\[ X(f) = \frac{\sin(\pi f \frac{T_c}{2})}{\sqrt{T_c \pi f}} e^{-j2\pi f \frac{T_c}{4}} - \frac{\sin(\pi f \frac{T_c}{2})}{\sqrt{T_c \pi f}} e^{-j2\pi f \frac{3T_c}{4}} \]

\[ = \frac{\sin(\pi f \frac{T_c}{2})}{\sqrt{T_c \pi f}} \left[ e^{-j2\pi f \frac{T_c}{4}} - e^{-j2\pi f \frac{3T_c}{4}} \right] \]  

(A.2)

The \(|X(f)|^2\) can then be obtained as the product of \(X(f)\) and its complex conjugate \(X^*(f)\), and with some algebraic manipulation it is possible to derive

\[ |X(f)|^2 = X(f) \cdot X^*(f) = \frac{\sin^2(\pi f \frac{T_c}{2})}{T_c \pi^2 f^2} \left[ e^{-j2\pi f \frac{T_c}{4}} - e^{-j2\pi f \frac{3T_c}{4}} \right] \cdot \left[ e^{j2\pi f \frac{T_c}{4}} - e^{j2\pi f \frac{3T_c}{4}} \right] \]  

(A.3)

The simplified expression for the autocorrelation can now be easily obtained as the inverse Fourier Transform of the expression (A.3), which leads to

\[ R_x(\tau) = A \left( \frac{\tau}{T_c/2} \right) - \frac{1}{2} A \left( \frac{\tau - T_c/2}{T_c/2} \right) - \frac{1}{2} A \left( \frac{\tau + T_c/2}{T_c/2} \right) \]  

(A.4)

where \(A \left( \frac{t}{T} \right)\) is the triangular function defined as

\[ A \left( \frac{t}{T} \right) = \begin{cases} 
1 - \frac{|t|}{T}, & |t| \leq T \\
0, & |t| > T 
\end{cases} \]

The approximated autocorrelation function described by Equation A.4 is shown in Figure A.2

Figure A.2. Approximated autocorrelation function for the Galileo BOC(1,1)
Appendix B

Circular and linear correlation performed by means of DFT operations

This appendix clarifies how it is possible to compute a circular and a linear correlation using DFT transformations of two digital sequences. Some basic properties of the digital sequences and their DFT transforms must be first introduced:

**basic properties**

Given the periodic sequence \( x_p \) defined as follows:

\[
x_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{jn \frac{2\pi}{N} k}
\]  

(B.1)

where \( X[k] \) is the DFT transform of the sequence \( x[n] \) of length \( N \) it is possible to derive the following relations:

\[
x_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_p[k] e^{jn \frac{2\pi}{N} k}
\]  

(B.2)

\[
X_p[n] = \sum_{k=0}^{N-1} x_p[n] e^{-jn \frac{2\pi}{N} k}
\]  

(B.3)
Circular correlation

The expression of the circular correlation by means of DFTs can be derived considering two sequences \( x[n] \) and \( h[n] \) both of length \( N \) and evaluating the IDFT of the sequence:

\[
W[k] = X[k]H[k] = X(e^{j\omega})X(e^{j\omega}) \big|_{\omega = 2\pi k/N} \tag{B.4}
\]

the expression of \( w[n] \) can be obtained from Equation (B.2) writing:

\[
w_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} X_p[k]H_p[k]e^{jn2\pi k/N} \tag{B.5}
\]

and expressing \( X_p[k] \) and \( H_p[k] \) by means of Equation (B.3) it is possible to write

\[
w_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} x_p[i]e^{-j\frac{2\pi}{N} ki} \sum_{q=0}^{N-1} h_p[q]e^{-j\frac{2\pi}{N} kq}e^{jn2\pi k/N} \tag{B.6}
\]

which is equivalent to

\[
w_p[n] = \frac{1}{N} \sum_{i=0}^{N-1} x_p[i] \sum_{q=0}^{N-1} h_p[q] \sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N} (n-i-q)}

= \frac{1}{N} \sum_{i=0}^{N-1} x_p[i] \sum_{q=0}^{N-1} h_p[q] \frac{e^{-j\frac{2\pi}{N} (n-i-q)N} - 1}{e^{-j\frac{2\pi}{N} (n-i-q)} - 1} \tag{B.7}
\]

Since \( w_p \) is periodic, it is possible to consider Equation (B.7) just in the interval \( 0 \leq n \leq N - 1 \). In this interval the term

\[
\frac{e^{-j\frac{2\pi}{N} (n-i-q)N} - 1}{e^{-j\frac{2\pi}{N} (n-i-q)} - 1}
\]

of Equation (B.7) is identical to \( N \) when \( q = n - i \) and equal to zero when \( q \neq n - i \), then it can be rewritten as

\[
w_p[n] = \sum_{i=0}^{N-1} x_p[i]h_p[n-i]
\]

This last relation is the circular correlation which can be then evaluated by means of DFTs and IDFT as

\[
w[n] = \text{IDFT} \{ \text{DFT} \{ x[n] \} \text{ DFT} \{ h[n] \}^* \}
\]

where the complex conjugate generalizes this expression to complex signals. Usually the circular correlation is denoted with the symbol "\( \otimes \)"
Linear correlation

The linear correlation can be obtained as a circular correlation using the zero padding technique. Considering the same two sequences $x[n]$ and $h[n]$ with the zero padding it is possible to build the following signals:

$$x_z = \begin{cases} 
  x[n] & \text{if } 0 \leq n \leq N - 1 \\
  0 & \text{if } N \leq n \leq 2N - 1
\end{cases}$$  \hspace{1cm} (B.8)

and

$$h_z = \begin{cases} 
  h[n] & \text{if } 0 \leq n \leq N - 1 \\
  0 & \text{if } N \leq n \leq 2N - 1
\end{cases}$$  \hspace{1cm} (B.9)

Remembering the definition of linear correlation

$$y[n] = \sum_{m=-\infty}^{+\infty} x[m] h[n - m]$$

and comparing $y[n]$ with $y'[n]$ obtained as

$$y'[n] = \sum_{m=-\infty}^{+\infty} x_z[m] h_z[n - m]$$

it is easy to verify the identity $y'[n] = y[n]$. Now, applying the circular correlation to $x_z$ and $h_z$ it is almost immediate to verify that

$$w[n] = x_z[n] \otimes h_z[n] = y'[n] = y[n]$$

It follows that the zero padding allows the use of the circular correlation to evaluate a linear correlation.
B – Circular and linear correlation performed by means of DFT operations
Appendix C

Analytical computation of the envelope detector output in case of complete alignment

In order to obtain analytically the expression of the envelope detector output in case of perfect code delay and Doppler alignment for a completely floating–point serial search scheme, it is necessary to consider Equations (8.4) and (8.6) and to evaluate the summations.

This appendix clarifies the analytical computation of the cited equations.

Expressing the cosine in the exponential form, it is possible to write

\[
\sum_{n=0}^{N-1} \cos(\omega n) = \frac{1}{2} \sum_{n=0}^{N-1} \left( e^{j\omega n} + e^{-j\omega n} \right) = \\
= \frac{1}{2} \sum_{n=0}^{N-1} e^{j\omega n} + \frac{1}{2} \sum_{n=0}^{N-1} e^{-j\omega n} = \\
= \frac{1}{2} \left[ \frac{1}{1 - e^{j\omega}} + \frac{1}{1 - e^{-j\omega}} \right] \\
= 1 - e^{j\omega N} - e^{-j\omega N} \\
= \frac{1}{2} \left[ \frac{\sin(\omega N/2)}{\sin(\omega/2)} + \frac{\sin((N-1)\omega/2)}{\sin((N-1)\omega/2)} \right]
\]

Collecting \( e^{\pm j\omega N/2} \) from the numerator and \( e^{\pm j\omega} \) from the denominator of the previous expressions, the sum of the cosines becomes

\[
\sum_{n=0}^{N-1} \cos(\omega n) = \frac{1}{2} \left[ e^{j\omega/2} (N-1) \sin(\omega/2) + e^{-j\omega/2} (N-1) \sin((N-1)\omega/2) \right]
\]
which finally leads to

\[
\sum_{n=0}^{N-1} \cos(\omega n) = \cos \left( \frac{\omega}{2} (N - 1) \right) \frac{\sin \left( \frac{\omega N}{2} \right)}{\sin \left( \frac{\omega}{2} \right)} = N \cos \left( \frac{\omega}{2} (N - 1) \right) D_N \left( \frac{\omega}{2} \right)
\]  

(C.1)

where \( D_N \left( \frac{\omega}{2} \right) \) is the Dirichlet function, plotted in Figure C.1 for different \( N \) values.

In an analogous way, the sum of the sine waves can be expressed as

\[
\sum_{n=0}^{N-1} \sin(\omega n) = \frac{1}{2j} \sum_{n=0}^{N-1} \left( e^{j\omega n} - e^{-j\omega n} \right) = \\
= \frac{1}{2j} \sum_{n=0}^{N-1} e^{j\omega n} - \frac{1}{2j} \sum_{n=0}^{N-1} e^{-j\omega n} = \\
= \frac{1}{2j} \left[ 1 - e^{j\omega N} \right] - \frac{1}{2j} \left[ 1 - e^{-j\omega N} \right]
\]

Collecting \( e^{\pm j\omega \frac{N}{2}} \) from the numerator and \( e^{\pm j\frac{\omega}{2}} \) from the denominator of the previous expressions, the sum of the sines becomes

\[
\sum_{n=0}^{N-1} \sin(\omega n) = \frac{1}{2j} \left[ e^{j\omega (N-1)} \frac{\sin \left( \frac{\omega N}{2} \right)}{\sin \left( \frac{\omega}{2} \right)} - e^{-j\omega (N-1)} \frac{\sin \left( \frac{\omega N}{2} \right)}{\sin \left( \frac{\omega}{2} \right)} \right]
\]

so that it is possible to write

\[
\sum_{n=0}^{N-1} \sin(\omega n) = \sin \left( \frac{\omega}{2} (N - 1) \right) \frac{\sin \left( \frac{\omega N}{2} \right)}{\sin \left( \frac{\omega}{2} \right)} = N \sin \left( \frac{\omega}{2} (N - 1) \right) D_N \left( \frac{\omega}{2} \right)
\]  

(C.2)
Appendix D

Γ Distribution

Given a random variable \( Y \sim N(\mu, \sigma^2) \), the distribution of \( X = Y^2 \) can be evaluated by means of the square transformation property. From reference [38], the square transformation leads to a new density function of the form:

\[
f_X(x) = \frac{1}{2\sqrt{x}} \left[ f_Y(\sqrt{x}) + f_Y(-\sqrt{x}) \right] u(x) \tag{D.1}
\]

With \( u(x) \) the unitary echelon function:

\[
u(x) = \begin{cases} 
1 & x \geq 0 \\
0 & x < 0
\end{cases} \tag{D.2}
\]

For the gaussian case it is possible to write:

\[
f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2} \frac{(y-\mu)^2}{\sigma^2}} \tag{D.3}
\]

and the density of \( X \) becomes:

\[
f_X(x) = \frac{1}{2\sqrt{x}} \frac{1}{\sqrt{2\pi\sigma^2}} \left[ e^{-\frac{1}{2} \frac{(\sqrt{x}-\mu)^2}{\sigma^2}} + e^{-\frac{1}{2} \frac{(\sqrt{x}+\mu)^2}{\sigma^2}} \right] u(x) \tag{D.4}
\]

\[
f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2x}} e^{-\frac{1}{2} \frac{x+\mu^2}{\sigma^2}} \cosh \left( \frac{\sqrt{x}\mu}{\sigma^2} \right) u(x) \tag{D.5}
\]

When \( \mu = 0 \) equation (D.5) degenerates in:

\[
f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2x}} e^{-\frac{1}{2} \frac{x}{\sigma^2}} u(x) \tag{D.6}
\]

\[
f_X(x) = \frac{1}{\Gamma \left( \frac{1}{2} \right) (2\sigma^2)^{\frac{1}{2}}} x^{\frac{1}{2}-1} e^{-\frac{x}{2\sigma^2}} u(x) \tag{D.7}
\]
Where $\Gamma(\cdot)$ is the Euler’s gamma function, which is equal to $\sqrt{\pi}$ when the argument is equal to $\frac{1}{2}$. Equation (D.7) means that $X$ is distributed according to a $\Gamma$ law of parameters $\frac{1}{2}$ and $2\sigma^2$.

$$X \sim \Gamma\left(\frac{1}{2}, 2\sigma^2\right)$$  \hspace{1cm} (D.8)

When also the condition $\sigma = 1$ leads to

$$X \sim \chi^2(1)$$  \hspace{1cm} (D.9)

The chi-square distribution is a particular case of the $\Gamma$ law with parameters $\frac{1}{2}$ and 2 and it is obtained squaring a normal random variable.

**Basic property of the $\Gamma$ distributions**

Let $(X_i)_{i=1}^N$ be independent random variables of law:

$$X_i \sim \Gamma(\alpha_i, \beta)$$  \hspace{1cm} (D.10)

then $Y = \sum_{i=1}^N X_i$ is

$$Y \sim \Gamma\left(\sum_{i=1}^N \alpha_i, \beta\right)$$  \hspace{1cm} (D.11)
Appendix E

Marcum Q function

It is well known that, given $I$ and $Q$, two independent gaussian random variables with mean different from zero, the quantity

$$Z = \sqrt{I^2 + Q^2}$$ (E.1)

it is a random variable distributed according to a Rice distribution, with a probability density function (pdf) defined as

$$f_Z(z) = \frac{z}{\sigma^2} e^{-\frac{1}{\sigma^2}(z^2 + \alpha^2)} I_0 \left( \frac{2\alpha}{\sigma^2} \right) u(z)$$ (E.2)

where $I_0(\cdot)$ is the modified Bessel function of zero order, which has the expression:

$$I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{x \cos(\theta)} d\theta$$ (E.3)

By applying the squaring property introduced in Appendix D, the distribution of $Y = Z^2$ can be written as

$$f_Y(y) = \frac{1}{2\sigma^2} e^{-\frac{1}{\sigma^2}(y + \alpha^2)} I_0 \left( \frac{\alpha \sqrt{y}}{\sigma^2} \right) u(y)$$ (E.4)

and the below integral

$$P(V) = \int_V^{+\infty} \frac{1}{2\sigma^2} e^{-\frac{1}{\sigma^2}(y + \alpha^2)} I_0 \left( \frac{\alpha \sqrt{y}}{\sigma^2} \right) u(y) dy$$ (E.5)

expressed in the following form, after the variable substitution $t = \frac{y}{2\sigma^2}$

$$P(V) = \int_{2\sigma^2 T}^{+\infty} te^{-\frac{t^2 + \frac{\alpha^2}{2}}{2}} I_0 \left( \frac{\alpha t}{\sigma} \right) dt$$ (E.6)

with $T > 0$. 

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In reference [66], the so called Marcum Q functions are defined as

\[ Q(\alpha, \beta) = \int_{\beta}^{+\infty} x e^{-\frac{x^2 + \alpha^2}{2\sigma^2}} I_0(x\alpha) \, dx \]

and it can be numerically evaluated. With the introduction of this definition, Equation (E.6) becomes

\[ P(T) = Q_1\left(\frac{\alpha}{\sigma}, \sqrt{T} \right) \quad (E.7) \]

The above procedure can be extended to \( Y = \sum Z^2 \), which is the case of non coherent summations prior the envelope detector. It is here recall that the pdf of a summation of variables, can be derived as the convolution of the pdf of the single variables. However, the calculation can be simplified using the characteristic function defined as

\[ C(p) = \int_{0}^{+\infty} f_Y(x)e^{py} \, dx \]

The characteristic function of a sum of variables is

\[ C_Y(p) = \prod C_{Z^2}(p) \]

since

\[ C_1(p) = \int_{0}^{+\infty} e^{-y - \frac{y^2}{2\sigma^2}} I_0\left(2\frac{\alpha}{\sqrt{2\sigma}}\right) e^{2\sigma^2 py} \, dy = \frac{1}{2\sigma^2 + 1} e^{-\frac{\alpha^2}{2\sigma^2}} e^{\frac{\alpha^2}{2\sigma^2} \cdot \frac{1}{2\sigma^2 + 1}} \]

it is possible to obtain the final expression for the pdf of \( Y \) as the inverse transform of \( \hat{C}_k(p) = C_1(p)^k \)

\[ C_k(p) = \frac{e^{-\frac{\alpha^2}{2\sigma^2}} \frac{1}{2\sigma^2 + 1}}{(2\sigma^2 p + 1)^k} e^{\frac{\alpha^2}{2\sigma^2} \cdot \frac{1}{2\sigma^2 + 1}} \]

and the final expression for \( f_Y^k(y) \) is

\[ f_Y^k(y) = \frac{1}{2\sigma^2} \left( \frac{y}{k\alpha^2} \right)^{k-1} e^{-\frac{y^2}{2\sigma^2} - \frac{k\alpha^2}{2\sigma^2}} I_{k-1}\left(\frac{\alpha}{\sigma^2} \sqrt{ky}\right) \quad (E.8) \]

a more detailed prove of the derivation of the above equations can be found in [66].

The integral \( P(T) = \int_{T}^{+\infty} f_Y^k(y) \, dy \) can now be expressed, after the substitution \( t^2 = \frac{t}{\sigma^2} \), as

\[ P(V) = \int_{\sqrt{V}}^{+\infty} f_Y^k(y) \, dy = \int_{\sqrt{\frac{V}{\sigma^2}}}^{+\infty} k \frac{1}{\alpha^2} \left( \frac{t\sigma}{\alpha} \right)^{k-1} t e^{-\frac{1}{2}(t^2 + \frac{k\alpha^2}{\sigma^2})} I_{k-1}\left(\frac{\sqrt{V\alpha^2}}{\sigma^2} \right) \, dt \quad (E.9) \]
and finally, introducing the $k^{th}$ order Marcum Q function, addressed as "Generalized Q function" as it follows

$$Q_k(\alpha, \beta) = \frac{1}{\alpha^{k-1}} \int_{\beta}^{+\infty} x^k e^{-\frac{1}{2}(x^2+\alpha^2)} I_{k-1}(\alpha x) \, dx$$

which does not appear possible to be expressed in terms of a finite number of known function, and therefore it has to be evaluated numerically.
$E$ – Marcum Q function
Bibliography


